Objective

Experimentation with self-calibration algorithms for rotating cameras has shown that whilst pin-cushion radial lens distortion causes graceful failure, barrel distortion causes a dramatic breakdown. Why is this?

Motivation

- Cameras with moveability which can only rotate (e.g. on a tripod) are of considerable practical importance, being widely used in surveillance applications and television broadcasts.
- Whilst automatic calibration of such cameras has received great attention (e.g. Hartley - ECCV 1994, Zhang - IJPR 1996, Hartley - BMVC 1997, Agapito et al. BMVC 1998), few methods have been paid to radial distortion.
- We have found radial distortion to be a hindrance for accurate self-calibration. We examine why this is so and suggest possible routes towards automatic correction.

Preamble: radial distortion

For an image centre around the origin, distorted and undistorted points are related by:

\[
X_2 = 
\begin{pmatrix}
X_0 \\
Y_0
\end{pmatrix}
\begin{pmatrix}
1 & \kappa x_0^2 \\
0 & 1
\end{pmatrix}. 
\]

- Images containing negative distortion, \( \kappa < 0 \), exhibit barrel distortion and such images are corrected by applying positive distortion (and vice versa for pin-cushion images, \( \kappa > 0 \)).

This implies the following assumptions and approximations:

(i) only the first distortion parameter is used;
(ii) the distortion is centered around the image centre;
(iii, iv) zero skew and unity aspect ratio, so that the distortion can be measured and corrected in image coordinates.

Distorted rotating camera geometry —

An assumption in self-calibration algorithms for rotating cameras is that the 3D lines connecting marked features all intersect at the same fixed point in space — the rotation centre. We make two key observations:

- Radial distortion causes problems because these reconstructed "match-lines" no longer intersect at the rotation centre.
- Pin-cushion distortion results in intersections that are close to the rotation centre, but barrel distortion causes a wide spread of intersections.

How much distortion is tolerable?

An alternative way of visualising the effect of radial distortion is as a warping of the image plane.

- Barrel distortion bends the image plane closer to a spherical projection.
- Spherical projection, where the image surface lies on the rotation sphere, provides no information about focal length or principal point.

This can be seen from 3D relations for the circular and line projections:

\[
\begin{align*}
\kappa &= \frac{f_0 + \hat{f} \theta}{f_0 + \frac{1}{\theta^2}}, \\
\kappa &= \frac{f_0}{1 - \kappa \theta^2}.
\end{align*}
\]

- For some value of distortion \( \kappa \), the image plane closely matches the spherical projection.
- The distortions along the circular and line projections are \( \kappa_c = f \theta \) and \( \kappa_t = f \tan \theta \), so that \( \kappa_c = f \tan \theta \).

Effect on self-calibration

Experimental data from a real calibration algorithm shows that:

- Pin-cushion distortion causes underestimation of the focal length with a steadily increasing bias (error bars show one standard deviation over 100 trials).
- Barrel distortion causes rapid increase in both mean and standard deviation of the estimated focal length, before becoming completely unreliable by \( \kappa = -8 \times 10^{-5} \), as predicted.

Recovering \( \kappa \) automatically

Unless \( \kappa \) can be recovered automatically, existing self-calibration algorithms for rotating cameras will give unreliable results in the presence of negative distortion, and reliable but biased results when the distortion is positive. A fully automatic calibration therefore also needs to determine and remove distortion.

One possible method is to assume the distortion varies smoothly over time. Polynomial patches can then be fitted to the distortion by minimizing the residual of the inter-image homographies (a homography cannot correctly model the distortion).

(Footnote: A second more robust method uses the property that positive distortion gives a reliable but biased calibration with correct relative focal length:

(i) Distort all images by a large positive factor.
(ii) Self-calibrate (relatively unbiased if distortion is positive).
(iii) Solve for distortion against focal length (function shape is well known).
(iv) Correct distortion in original images.

(v) Re-calibrate from corrected images.)