Component isolation for multi-component signal analysis using a non-parametric gaussian latent feature model

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A challenge in analysing non-stationary multi-component signals is to isolate nonlinearly time-varying signals especially when they are overlapped in time and frequency plane. In this paper, a framework integrating time-frequency analysis-based demodulation and a non-parametric Gaussian latent feature model is proposed to isolate and recover components of such signals. The former aims to remove high-order frequency modulation (FM) such that the latter is able to infer demodulated components while simultaneously discovering the number of the target components. The proposed method is effective in isolating multiple components that have the same FM behavior. In addition, the results show that the proposed method is superior to generalised demodulation with singular-value decomposition-based method, parametric time-frequency analysis with filter-based method and empirical model decomposition base method, in recovering the amplitude and phase of superimposed components.

1. Introduction

In many applications encountered in practice, non-stationary multi-component signals consist of various non-linearly frequency-modulated (FM) components. The latter convey important information concerning the non-stationary process that assumedly generates the signals. A critical signal processing task is to isolate FM components, where this is complicated by the complex time-frequency structures of the signals; e.g., the micro-Doppler signatures of radar echo corresponding to components of moving target are intersected in time-frequency domain; the characteristic frequencies of rotatory machine and their harmonics are inseparable in frequency domain undergoing non-stationary operating process; the frequencies and harmonics of underwater mammal calls vary nonlinearly and are inseparable in frequency domain. In general, the isolated non-stationary components as important features are useful in both target/novelty detection, condition monitoring, etc.

Empirical mode decomposition (EMD) is known for decomposing a signal into a series of intrinsic mode functions (IMF) [6], each of which is assumed to be a mono-component. However, EMD is incapable of dealing with intersected components or broad-bandwidth nonlinear FM components. Filter- or mask-based methods in the time-frequency domain could cope with such components [1], though a substantial challenge is to determine bandwidth or masking size to capture local features. A trade-off of using filters or masks lies in recovering the intersected components; i.e., one is fully recovered while the other will be broken down and cannot be recovered at the point at which components cross. Widely-studied component
separation methods such as the Hough transform [2], singular value decomposition [3], and energy separation [4] perform well in the case of intersected stationary components, though the number of the target components often needs to be defined in advance.

The non-parametric Gaussian latent feature model (NGLF) [12–16] is known for its capacity to infer latent features without prior knowledge. The components of a multi-component signal can be considered as being latent features of the signal; the NGLF thereby holds significant potential for improved component isolation. The NGLF is superior to independent component analysis [7–9] and non-negative matrix factorisation [10], which are two well-studied models for automatic component isolation, in the case that the number of the latent features is unknown. However, the latter methods assume that the latent feature is static in the observations, while time-frequency patterns of FM components vary with time. A challenge exists in how to apply the NGLF to the identification of FM components. A straightforward solution is to project the FM components into a space where the components are stationary so that the NGLF is able to resolve the latent features.

In this paper, we propose an integrated framework that combines (i) a time-frequency analysis-based demodulation and (ii) the NGLF. The demodulation technique assumes a frequency-modulated signal model with the phase modulating by a high-order function of time [17,18]. To acquire the parameters of an arbitrary phase function, we adapt the parametric time-frequency transform (PTFT) [5]. The latter was proposed as a time-frequency domain analysis method that is capable of using an arbitrary phase transform kernel to capture the local time-frequency pattern of a non-stationary signal, and to obtain a time-frequency representation with enhanced energy concentration. An iterative parameter estimation scheme associated with the PTFT provides the basis for demodulating various multi-component signals. In [23], a combination of the PTFT and simple band-pass filter was applied for component extraction, though the filter resulted in attenuation at the edges of components and amplitude distortion at the cross-point between intersecting components. In [21], the PTFT was integrated with singular value decomposition (SVD), but we note that the SVD strongly relies on the accuracy of the estimation of the demodulation. In addition, it can neither deal with multiple components with the same frequency modulation nor with local components. In [22], a joint-refinement post-processing step was used to recover the amplitude of intersected components, though it assumed that the component is known and the estimation for all components cannot reach the best simultaneously. The framework described in this paper provides alternative solutions to cope with the above limitations in component isolation.

In the proposed framework, the PTFT aims to remove the high-order FM in components so that the NGLF is able to infer the demodulated components automatically without defining the number of the components. It is shown that the proposed method is effective in isolating multiple components with the same FM behavior at one time. In addition, the results show that the proposed method is superior to generalised demodulation with singular-value decomposition-based isolation, parametric time–frequency analysis with filter-based isolation and empirical model decomposition base isolation, in recovering the amplitude and phase of superimposed components.

The remainder of this paper is organised as follows: Section 2 introduces a signal model and the underlying principles of the NGLF and PTFT; Section 3 provides the details of the proposed framework and a demonstration on an exemplar signal. Section 4 provides analysis results of applying the proposed method with both simulated and experiment signals. Finally, Section 5 draw conclusions.

2. Theoretical background

2.1. Signal model

We assume a multi-component signal $S(t) \in \mathbb{R}^1$ with $L$ components $s(t)$,

$$S(t) = \sum_{l=1}^{L} s_l(t)$$  

with

$$s(t) = A(t) \exp\{-i\phi(t)\}$$  

where $\sqrt{t} = -1$, $A(t)$ and $\phi(t)$ are time-varying amplitude and phase, respectively. Eq. (2) is an analytic form of signal components. Instantaneous frequency (IF) is a measurement of time-frequency pattern defined for non-stationary signal as,

$$IF(t) = \frac{1}{2\pi} \frac{d}{dt} \phi(t)$$  

which assumes that the phase function $\phi(t)$ is differentiable. Noted that this definition is for mono-component signals, it is also valid for individual component of multi-component signals.

A signal with finite time support can be cast into a set of segments with overlap. For example, the multi-component signal $S(t)$ is divided into a set of overlapped segments with the length of $D$; i.e., $X = \{x^1, \ldots, x^N\}, x^i \in \mathbb{R}^D$.

We assume that a component set $A = \{a^1, \ldots, a^D\}, a^i \in \mathbb{R}^D$ is responsible for generating the data $X$,

$$X = W^A$$
where $W$ is a binary matrix with $w_{ij} = [0, 1]$ denoting if the $i$th example contains the $j$th feature. $K$ stands for the number of features and $K > L$. The equality holds if and only if the components are stationary.

### 2.2. Parametric time-frequency transform

The parametric time-frequency transform (PTFT) is a general formula that uses a kernel function to transform a time-domain signal into the time-frequency domain [20]. An advantage of the PTFT is that it applies adaptive kernels that approximate time-frequency patterns of analysed signals to improve the readability of their time-frequency representations. It is especially effective for non-stationary signals with various shapes of IF trajectory in the time-frequency domain.

The PTFT is defined as

$$PTFT(t, \omega; P) = \int_{-\infty}^{\infty} \tilde{s}(\tau)h(\tau - t)\exp\{-i\omega \tau\}d\tau$$

with

$$\tilde{s}(\tau) = s(\tau)\Phi^{\delta}(\tau; P)\Phi^{\delta}(\tau; t_0)$$

$$\Phi^{\delta}(\tau; P) = \exp\left\{-i\int K(\tau; P) d\tau\right\}$$

$$\Phi^{\delta}(\tau; t_0) = \exp\{i\tau K(t_0; P)\}$$

where $\Phi^{\delta}(\tau; P)$ and $\Phi^{\delta}(\tau; t_0)$ are frequency rotation and shifting operators, respectively; $\omega$ is frequency; $h(t)$ is a window function; $K(\tau; P)$ denotes the kernel function that can be integrated with the time given the parameter $P$, $t_0$ is the central time within an interval of evaluation $\tau$. The goal of the PTFT is to apply the frequency rotation operator, $\Phi^{\delta}(\tau; P)$, to remove the time variation of frequency across all frames of the signal within interval $\tau$, and then use the frequency shift rotation operator, $\Phi^{\delta}(\tau; t_0)$, to carry the time-invariant frequency spectrum back to the centre of the each frame.

As counterparts of the PTFT, non-parametric time–frequency transforms with a signal-independent window function, (e.g., the short-time Fourier transform and the continuous wavelet transform), result in smearing of energy in the time-frequency representation in the case of a non-stationary signal. As a result, the readability of the time-frequency representation will be reduced due to the unsatisfied energy concentration. On the other hand, the PTFT offers twofold benefits: (1) concentration the otherwise-smeared energy in the grid of time-frequency representation; (2) optimising parameters in terms of energy concentration in the rotated frequency spectrum.

### 2.3. Non-parametric Gaussian latent feature model

The non-parametric Gaussian latent feature (NGLF) model is a statistical model that assumes that the examples are generated by a Gaussian distribution with covariance and mean given by a linear combination of latent features and binary weights. The latent features and binary weights are assumed to have priors that are a Gaussian process (GP) and Indian buffet process (IBP), respectively. The purpose of the model is to infer the posterior of the binary weights given the data, meanwhile the latent features are obtained given the data and the binary weights. In our study, the latent features correspond to the components and noise, while the binary weights denote the presence or absence of those components.

The model assumes that a $d$-dimensional vector of an example $x_i$ is generated from a Gaussian distribution with mean $w_{iA}$ and covariance matrix $\sum_{i} = \sigma^2_I$, where $w_i$ is a $K$-dimensional binary vector, and $A$ is a $K \times D$ matrix (as shown in Fig. 1).

According to Bayes’ theorem, the posterior distribution over $W$ can be obtained via

$$p(W|X, \sigma_X, \sigma_A) \propto p(X|W, \sigma_X, \sigma_A)p(W)$$

![Fig. 1. Graphical model for the NFL [19].](image)
where \( \sigma_X \) is the covariance of observation matrix \( X \). The first term in the right-hand side of Eq. (7), the likelihood of \( p(X|W, \sigma_X, \sigma_A) \), is computed by marginalising the unknown feature \( A \),

\[
p(X|W, \sigma_X, \sigma_A) = \int p(X|W, A, \sigma_X)p(A|\sigma_A) dA
\]

(8)

where the probability of \( X \) given \( W, A \), and \( \sigma_X \) is modelled as a multivariate Gaussian distribution as aforementioned. The probability of \( A \) given \( \sigma_A \) is also a multivariate Gaussian distribution, \( MN(0, \sigma_A^2I) \). Therefore, Eq. (8) is computed via

\[
p(X|W, \sigma_X, \sigma_A) = \frac{1}{(2\pi)^{NWR_2} \sigma_X^{-N} \sigma_A^{MN} |H|^{R/2}} \exp \left\{ -\frac{1}{2\sigma_X^2} \text{tr}\left( X' \left( I - WH^{-1}W^t \right) X \right) \right\}
\]

(9)

with

\[
H = W^tW + \sigma_A^2I
\]

(10)

where \( \text{tr}() \) is the trace of a matrix.

The prior probability of \( W \), the second term in the right-hand side of Eq. (7), is assumed to follow the IBP, which allows the matrix to have an unbounded number of columns. A generative process for a binary weight matrix is defined as,

\[
\pi_j|\alpha \sim \text{Beta} \left( \frac{\alpha}{R}, 1 \right) \quad w_j|\pi_j \sim \text{Ber} \left( \pi_j \right)
\]

(11)

Each \( w_j \) follows a Bernoulli distribution and is independent of all other assignments, conditioned on \( \pi_j \). The \( \pi_j \) follows a Beta distribution and is generated as being independently conditioned on \( \alpha \). By interpreting the generative process using an IBP, the probability of each entry in the binary matrix \( p(w_{ij}|w_{-ij}) \) can be evaluated as,

\[
p(w_{ij} = 1|w_{-ij}) = \frac{m_{ij} + \frac{\alpha}{R}}{N + \frac{\alpha}{R}}
\]

(12)

where \( m_j = \sum_{i=1}^N w_{ij} \). The marginal probability of the binary matrix \( W \) is

\[
p(W|\alpha) = \prod_{k=1}^K \frac{\Gamma(m_k + \frac{\alpha}{R}) \Gamma(N - m_k + 1)}{\Gamma(N + 1 + \frac{\alpha}{R})}
\]

(13)

where \( \Gamma() \) is the gamma function.

In aggregate, the full conditional distribution for \( w_{ik} \) is given as

\[
p(w_{ij}|X, W_{-ij}, \sigma_X, \sigma_A) \propto p(X|W, \sigma_X, \sigma_A)p(w_{ij}|W_{-ij})
\]

(14)

As the IBP assumes that the rows and columns are exchangeable, the generative process results in a group of binary matrices; these matrices share a unique left-order-form (lof) matrix which rearrange the rows and columns to the top and left according to the number of non-zero entries. The probability of a particular lof-equivalence class of binary matrices, \( [W]_h \), is

\[
p([W]_h) = \frac{K!}{2^{N-1}} \prod_{k=1}^K \frac{\Gamma(m_k + \frac{\alpha}{R}) \Gamma(N - m_k + 1)}{\Gamma(N + 1 + \frac{\alpha}{R})}
\]

(15)

where \( K_h \) is the count of the number of features with full history \( h \). The history of feature \( j \) for object \( i \) is defined to be \( (w_{ij}, \ldots, w_{(i-1)j}) \), while the full history of feature \( j \), without a specified object, is referred to as \( (w_{ij}, \ldots, w_{Nj}) \). Taking the limit for this probability of \( W \) as \( K \to \infty \) as

\[
\lim_{K \to \infty} p([W]_h) = \frac{\alpha^{K-1}}{2^{K-1} K!} \exp \left( -\alpha H_N \right) \prod_{k=1}^K \frac{(N - m_k)! (m_k - 1)!}{N! K_h!}
\]

(16)

where \( K_h \) is the number of columns for which \( m_k > 0 \) and \( H_N = \sum_{j=1}^N \frac{1}{j} \) is the Nth harmonic number. Considering an infinite number of features, the probability of \( X \) given \( W, \sigma_X \), and \( \sigma_A \) changes \( W \) and \( K \) in Eq. (9) to be \( W_* \) and \( K_* \), where \( W_* \) is the corresponding submatrix of the active features.

To perform inference with \( W \), a heuristic Gibbs sampling algorithm is used to draw examples via Eq. (14), using Eqs. (12) and (9) to compute \( p(w_{ij}|w_{-ij}) \) and \( p(X|W, \sigma_X, \sigma_A) \), respectively. Here, \( \sigma_A \) and \( \sigma_X \) are sampled via a Metropolis-Hastings step; the acceptance of proposed values is determined by evaluating \( p(X|W, \sigma_X, \sigma_A) \) from Eq. (9). For each new component,
\[ p(X|W, \sigma_X, \sigma_A) \] is also computed, and where the prior on the number of new components is Poisson(\( \lambda \)). The latent features are calculated as,

\[ E[A|W, X] = \left( W^TW + \frac{\sigma^2_A}{\sigma^2}I \right)^{-1} W^T X \]  

(17)

In aggregate, the NGLF model is the probabilistic model with the prior of IBP that allows the number of the latent features to be increased. The term of “non-parametric” means that one does not need to specify the number of the latent features, which will be learned by the non-parametric model from the data. We note that the complexity of such model can be varied with the data: the more latent features, the more complex the model will be.

3. Component isolation framework

3.1. Stationary component isolation

The NGLF aims to reveal latent features that are globally stationary. The underlying principle of the NGLF for component isolation is illustrated in Fig. 2. For a two-component signal with one stationary component, the time series is first cast into a series of sequentially overlapped segments. Next, an NGLF model takes the segments as input and obtains latent features. The obtained first latent feature corresponds to the stationary component. The other components may contain different segments of the non-stationary component and noise. This is because the NGLF considers non-stationary components to be local.
features. Then, the stationary component can be reconstructed using the first latent feature, meanwhile the other component can be obtained by removing the former from the original signal.

If there are multiple stationary components exist, the NGLF model will consider them as being one latent feature due to the co-occurrence. In this case, these co-occurred stationary components will be extracted from the analysed signal simultaneously. Such characteristics can also be used to distinguish stationary components from others; i.e., close-spaced quasi-stationary components and modulated noise.

### 3.2. Non-stationary component isolation

To isolate non-stationary components from a signal without stationary components, the PTFT and an associated parameter estimation scheme are used to pre-process the considered signal. The kernel function of the PTFT can be (for example) a polynomial, a Fourier series, or spline functions. In the investigation described in this paper, a polynomial kernel is used, which is given by,

\[ K(t; p_0, \ldots, p_m) = p_0 + p_1t + p_2t^2 + \ldots + p_mt^m \]  

where \( m \) is the order of the polynomial function. With the obtained time-frequency representation \( \text{PTFT}(t, \omega; P) \), the IF of the signal \( s(t) \) is estimated by evaluating the peak along the time axis,

\[ \text{IF}(t) = \arg \max_t |\text{PTFT}(t, \omega; P)|^2 \]  

The parameter estimation scheme iteratively approximates IF via Eqs. (19) and (18). This will only be accurate when the IF is well approximated. The time-frequency representation will be gradually improved when the estimated parameters become more accurate. Ideally, the estimated parameters enable the targeted component to be demodulated completely, though the parameter estimation process corrupted by noise in the original signal and interfered components that are intersected or which have high energy, typically results in incomplete demodulation. In this case, more than one of the latent features obtained by the NGLF model might include the considered component. Correspondingly, the latent features containing the unwanted components within the latent feature set need to be deleted. It is assumed that there are three types of the latent features: (1) the latent feature containing a stationary signal, which has narrow bandwidth and higher power; (2) the latent feature containing a non-stationary component, which has wide bandwidth and lower power; and (3) the latent feature containing both the stationary and non-stationary components, whose spectrum is either overlapped or non-overlapped. It is assumed that the non-stationary component and noise have much less power than the interested component in frequency domain.

Given the fact that spectrum of stationary signal is highly concentrated, a ratio describing the spectrum concentration is defined as the decreased power within a specified bandwidth over the summed power throughout the entire spectrum, which is given by

\[ R = \frac{\max_{\omega} F_{\omega}^a[\omega] - 0.5(F_{\omega}^a[\omega_p + 0.5b] + F_{\omega}^a[\omega_p - 0.5b])}{\sum_{\omega} F_{\omega}^a[\omega]} \]  

where \( F_{\omega}^a[\omega] \) denotes frequency spectrum of the latent feature \( a^\beta \), \( \omega_p \) and \( b \) stand for the frequency with peak power in the frequency spectrum, i.e., \( \omega_p = \arg \max_{\omega} F_{\omega}^a[\omega] \), and the bandwidth, respectively. The cut-off frequencies used for determining the bandwidth are recommended to be \( \omega_p \pm 0.5b \), where the bandwidth can be ranged between 1 and 5 Hz depending on

![Fig. 3. Component waveforms and TFR of Example 1: (a) waveform of \( s_1, s_2 \) and \( s_3 \) and (b) time-frequency plot of \( S \).](image-url)
both the frequency resolution and the accuracy of IF estimation, the latter of which would influence the accuracy of the demodulation. The higher the ratio, the more concentrated the spectrum. Thresholding the ratio is used to select the latent features of the first type from the other two. It is worth noting that thresholding the ratio is a separate post-feature-selection step, we therefore consider it to be irrelevant to the use of the term “non-parametric Gaussian latent feature model”.

Algorithm 1. The proposed component isolation framework

Data: $S(t)$, window length $d$, maximum sampling times $T_{max}$, terminating threshold $\delta$, threshold of ratio $\epsilon$

Result: Reconstructed components and residual

Initialization $i = 1$, $IF_0 = 0$, $P_0 = 0$, $\sigma_X$, $\sigma_A$;

while $|IF_i(t) - IF_{i-1}| < \delta$, $rS(t) = S(t)$ do

perform the PTFT via Eq. (5) and (6);

estimate IF of $s_i(t)$ via Eq. (19);

fit Eq. (18) for the estimated IF;

$i = i + 1$;

obtain kernel parameter $P_i$;

end

Demodulate $rS(t)$ using $P$ to obtain $S_d(t)$;

Cast $S_d(t)$ into $X$;

while $st < T_{max}$ do

Sample $\sigma_A^{(0)}$ using Metropolis-Hastings;

Sample $\sigma_X^{(0)}$ using Metropolis-Hastings;

Sample binary matrix $W^{(0)}$ via Eq. (12);

end

Compute $A$ via Eq. (17);

Select latent features with $R > \epsilon$ via Eq. (20), $A \rightarrow A^-$, $W \rightarrow W^-$;

Reconstruct component with $A^-$ and $W^-$, and $P$, $s_i(t) \rightarrow \tilde{s}_i(t)$;

$rS(t) \leftarrow rS(t) - \tilde{s}_i(t)$;

Repeat until no more intersected components remain in $rS(t)$;

The details of the proposed framework are provided in Algorithm 1. For stationary component isolation, the first loop for estimating phase parameter should be omitted. We note that the NGLF model is also able to isolate an asymptotic component whose amplitude is assumed to change more slowly than frequency.

For non-stationary components, the phase parameters are estimated via the PTFT-based iterative parameter estimation method, which are then used for demodulation as described earlier. The demodulated signal is cast as being a data matrix of segments using a sliding window. Each row of the matrix is considered as being an example observation. The NGLF is applied to the data matrix and obtains its weight matrix and latent features via Metropolis-Hastings and heuristic Gibbs sampling. Only those latent features containing the demodulated component are selected by evaluating the ratio in the frequency domain. Accordingly, the dimension of weight matrix is reduced. The selected latent features and binary matrix are then used for reconstructing the demodulated component, which is finally remodulated via the parameters estimated for demodulation. The process will be repeated until no more intersected components remain in the residual of the signal.

4. Analysis and discussion

In this section, we present results of analysis using three simulated signals and an experimental signal to illustrate the performance of the proposed method. All the simulated signals are contaminated by additive Gaussian noise with a signal-to-noise rate of 5 dB. For the comparison purpose, a BPF filter based component isolation, a variant of the ensemble EMD (complete ensemble empirical mode decomposition (CEEMD)) and a generalised demodulation combining the SVD (GD-SVD) are considered. In the following analysis (if not otherwise specified), the GD-SVD selected the first two singular values and the CEEMD selected the IMFs that highly correlated with the considered components, respectively.

4.1. Example 1

A 3-component signal is considered,

$$S(t) = s_1(t) + s_2(t) + s_3(t)$$  \hspace{1cm} (21)

with

$$s_1(t) = 0.5 \cos(2\pi t^2)$$ \hspace{1cm} (22)
Fig. 4. Component isolation for Example 1: (a) $s_1(t)$ reconstructed by GD-SVD; (b) TFR of the residual subtracting the reconstructed $s_1$ in (a); (c) $s_3$ obtained by CEEMD; (d) TFR of the residual given the obtained $s_3$ in (c); (e) $s_3$ obtained by BPF; (f) TFR of the residual subtracting the obtained $s_3$ in (e); (g) $s_3$ obtained by PTFT-NGLF; and (h) TFR of the residual subtracting the obtained $s_3$ in (g).
\begin{align}
  s_2(t) &= \cos[2\pi(12t - t^2)] \\
  s_3(t) &= 0.6 \cos(2\pi 6t)
\end{align}

where the IFs of the first two components are linear functions of time; i.e., 2t and 12 \(- 2t\), respectively. The proposed method is compared with linear filtering. The bandwidth of the filter is set to 5 \(- 7\) Hz. Fig. 3(a) shows the three-component waveforms. Fig. 3(b) shows the time-frequency representation (TFR) of the multi-component signal with the IF trajectories of the three components intersecting at \(t = 3\) s.

Fig. 4 shows the waveform of the isolated \(s_1\) and the TFR of the residual signal obtained by the GD-SVD, CEEMD, BPF and the proposed method (PTFT-NGLF), respectively. The results show that the PTFT-NGLF yields better performance in isolating stationary component from the non-stationary multicomponent signal when compared to the other methods considered (see Fig. 4(g) and (h)). Specially, the residual at the cross-point is preserved, leading to accurate reconstruction of the other two components sequentially (see Fig. 4(h)). The CEEMD performed the worst because it is essentially an adaptive BPF and suffers when the components are overlapped in the time–frequency plane (see Fig. 4(c) and (d)). The GD-SVD yielded lower accuracy in component reconstruction before the cross-point, where a pseudo-component was introduced in the residual (see Fig. 4(a) and (b)). This is likely because the phase coupling at the cross-point influenced the SVD or the second singular values might contribute to the inaccurate reconstruction. The BPF obtained the filtered component with attenuation at both ends and summed power of the three components at the cross-point, where the amplitudes of the other components cannot be recovered (see Fig. 4(e) and (f)).

4.2. Example 2

We consider a 3-component signal with components,

\begin{align}
  s_1(t) &= \cos[2\pi(2t + 0.7t^2)] \\
  s_2(t) &= \cos(2\pi 10t) \text{ where } s_2 = 0 \text{ if } t > 3.5 \text{ s} \\
  s_3(t) &= \cos(2\pi 4t)
\end{align}

where \(s_1(t)\) is a chirping component with a chirping rate of 1.4 Hz/s, \(s_2(t)\) is a local stationary component of 10 Hz from 0 s to 3.5 s, and \(s_3\) is a global stationary component of 4 Hz.

Fig. 5(a) and (b) shows the every component waveform and the TFR of the considered multi-component signal, respectively. It is shown that \(s_1\) is intersected with \(s_3\) at \(t = 1.5\) s. Fig. 5(c)-(h) plots the TFR of the separated components by the GS-SVD, CEEMD and PTFT-NGLF. The results of the BPF are now shown because it performed the same as in the Example 1 except that two BPFs were required in this example. The PTFT-NGLF substantially outperforms the other two methods, and demonstrates satisfied reconstruction of the two stationary components (see Fig. 5(g) and (h)). In Fig. 5(h), it is noticed that a Gabor-shaped component of 10 Hz at \(t = 3.5\) s remains because of phase coupling of \(s_2\) at \(t = 3.5\) s. Fig. 5(c) displays the local component \(s_2\) with the worst time localisation. This is likely because the SVD is global decomposition which attempts to decompose the local feature to be the global feature. On the other hands, the CEEMD failed to isolate the two stationary components at the same time, and distorted \(s_1\) at lower frequency. This is likely because the CEEMD is a global decomposition and essentially performs the same as a high-pass or low-pass filter with the cut-off frequency of 5 Hz.

4.3. Example 3

Here we consider a 2-component signal consisting of two nonlinear frequency-modulated (FM) components,

\begin{align}
  S(t) &= s_1(t) + s_2(t) \\
  s_1(t) &= \cos[2\pi(2t + 2t^2 - 0.2t^3)] \\
  s_2(t) &= 0.05 \cos[2\pi(12t - t^2)]
\end{align}

where \(s_1(t)\) is a polynomial phase signal with the IF of \(2 + 4t - 0.6t^2\), and \(s_2(t)\) is a chirping component with a chirp rate of \(-2\) Hz/s. Fig. 6(a) shows these two components are overlapped in the time–frequency domain at around \(t = 2\) s. As the amplitude of \(s_1(t)\) is the higher of the two components, we first isolate this component. Fig. 6(a) and (b) provides the component waveforms and TFR of the multi-component signal, respectively. Fig. 6(c)-(h) shows the results obtained by the GD-SVD, CEEMD and PTFT-NGLF. The BPF based method is omitted because that the two components are inseparable in frequency domain. Generally, the PTFT-NGLF performs the best, followed by the GD-SVD. Both methods introduced the small proportion of \(s_2\) into the isolated \(s_1\) at lower frequency. This is because the chirping rate of \(s_2\) is similar with \(s_1\) locally. The PTFT-NGLF yields the \(s_1\) with lower power at the cross-point (see Fig. 6(h)). This is likely because the \(s_1\) was obtained by
Fig. 5. Component isolation for Example 2: (a) component waveforms (b) TFR of the multi-component signal; (c) decomposed $s_2$ and $s_3$ by GD-SVD; (d) decomposed $s_1$ by GD-SVD; (e) decomposed $s_2$ and $s_3$ by CEEMD; (f) decomposed $s_3$ by CEEMD; (g) decomposed $s_2$ and $s_3$ by PTFT-NGLF; and (h) decomposed $s_1$ by PTFT-NGLF.
Fig. 6. Component isolation for Example 3: (a) component waveforms (b) TFR of the multi-component signal; (c) decomposed $s_1$ by GD-SVD; (d) decomposed $s_2$ by GD-SVD; (e) decomposed $s_1$ by CEEMD; (f) decomposed $s_2$ by CEEMD; (g) decomposed $s_1$ by PTFT-NGLF; and (h) decomposed $s_2$ by PTFT-NGLF.
subtracting the reconstructed $s_2$ from the multi-component signal, which reveals the phase coupling at the cross-point properly. As expected, the CEEMD performed as the filter with the cut-off frequency of 5 Hz (see Fig. 6(e) and (f)).

4.4. Example 4

Finally, a vibration time-series acquired from a hydroturbine in a hydroelectric power station was studied. Fig. 7(a) shows the TFR of the considered multi-component vibration signal when the hydroturbine operated on a stationary phase from 0 to 30 s, and shut down afterwards. Fig. 7(b) shows the reconstructed components obtained by the PTFT-NGLF, which contain the fundamental frequency throughout the entire process and high-order harmonics during stationary process. Fig. 7(c) and (d) shows the fundamental components isolated by the GD-SVD and CEEMD, respectively. Both of them captured the characteristic frequency at the stationary stage but lost information at the shut-down process.

It is worth mentioning that the IF estimation of signal component is a key step for the proposed method. However, searching TFR peaks along time axis is not robust for IF estimation especially with presence of strong noise. Alternatively, advanced ridge extraction based on image processing methods could improve IF estimation [24–26]. Given the errors in the IF estimation, the latent feature selection using the ratio in Eq. (20) allows quasi-stationary components to be isolated properly. For the proposed method, the interested component is assumed to be dominant and smooth in the multi-component signal, i.e., targeted component has higher power than non-targeted components and noise. Specially, in the case of intersected frequency-modulated and amplitude-modulated components, it is necessary to determine whether the estimated local ridges belong to the same component given the assumption on the smoothness, which will be the future work to be investigated.
5. Conclusion

In this study, we performed component isolation using a framework consisting of PTFT-based demodulation and the NGLF model. An advantage of the NGLF model is that it requires no information about the number of the components to be isolated, which enables the proposed method to isolate the components that potentially have the same higher-order FM characteristics. The proposed method is suitable for multi-component signals with intersecting stationary or non-stationary components. Simulations verify that the proposed method manages to recover the amplitude of isolated components at cross-points and that it can reveal phase-coupling effects. Analysis on real data shows that the proposed method is effective for isolating stationary components and for recovering non-stationary components. Computational efficiency can be improved by applying a variational inference procedure for the IBP or by using a sliding window without overlap. Future research will be directed towards the exploration of the NGLF model on time-series directly instead of assembling the segments as independent examples via pre-processing.

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