Parametric Time-frequency Analysis (TFA)

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OUTLINE

- Background
- Theory and methods
- Applications
Non-stationary signals

- Vibration signals
- Radar signals
- Non-stationary signals
- Bioelectric signal
- Seismic data and guided waves
- Electronic signal and speech
- Speech
Fourier transform-global transform

Stationary signal  Piece-wised stationary signal  Non-stationary signal

\[ s_1(t) = \sin(4\pi t) \]
\[ s_2(t) = \begin{cases} 
\sin(10\pi t) & t \in [0,1] \\
0.5\sin(22\pi t) & t \in (1,2] \\
\sin(50\pi t) & t \in (2,3] \\
0.5\sin(22\pi t) & t \in (3,4] 
\end{cases} \]
\[ s_3(t) = \sin(10\pi t + 2.5\pi t^2) \]
Non-stationary signal and time-frequency analysis

Instantaneous frequency (IF)

1937: Carson & Fry
1958: Ville

Group delay (GD)

\[ IF(t) = \frac{1}{2\pi} \frac{d\phi(t)}{dt} \]

\[ GD(f) = \frac{1}{2\pi} \frac{d\phi(f)}{df} \]

Controversy

- Uniqueness
  Shekel, Prestley, Huang
- Physical meaning
  Mandel, Cohen

Mono-component signal

Multi-component signal

IF
LGD

IF of cmp1
IF of cmp2
Time-frequency analysis (TFA)

Background:
TFA -> TFR (time-frequency representation) & TFD (time-frequency distribution)

Key issues of TFA:
Concentration improvement and cross-term suppression
TFAs

Short-time Fourier transform (STFT)
Joseph Fourier (1768-1830)
Dennis Gabor (1900-1979)

Wavelet transform
Alfréd Haar (1885-1933)
Jean Morlet
Ingrid Daubechies

Wigner-Ville distribution (WVD)
Eugene P. Wigner

Sparse decomposition
Stephane Mallat
Yves Meyer
Problem:
1&2-Poor concentration, incorrect instantaneous amplitude (IA)
3-Interference of cross-term
4-Poor characterization of IF

\[ s(t) = \sin \left[ 2\pi \left( 10t + \frac{5}{4}t^2 + \frac{1}{9}t^3 - \frac{1}{120}t^4 \right) \right] \]

\[ f(t) = 10 + 2.5t + t^2/3 - t^3/30 \]

1. STFT
2. Continues wavelet transform
3. WVD
4. Sparse decomposition
Signal model: \[ s(t) = A \exp \left[ j2\pi \left( f_0 t + \frac{c'}{2} t^2 \right) \right] \quad \text{IF: } f_0 + c't \]

Chirplet transform:

\[ CT_s(t, \omega; c) = \langle z(t), h_{t,\omega,c}(t) \rangle \]

Mother chirplet:

\[ h_{t,\omega,c}(t) = g_\sigma(\tau - t) \exp \left\{ j \left[ \omega(\tau - t) + \frac{c}{2} (\tau - t)^2 \right] \right\} \]

- **STFT**
- **Wavelet transform**
- **Chirplet transform**

Effect of chirplet parameter on Gaussian window function

- **TF resolution**
- **Section (t=120sec)**
CT and general parametric TFA

\[ CT_s(t, \omega; c) = \int_{-\infty}^{+\infty} z(\tau) g_\sigma^*(\tau - t) \exp \left\{ -j \left[ \omega(\tau - t) + \frac{c}{2} (\tau - t)^2 \right] \right\} d\tau \]

\[ CT_s(t, \omega; c) = A(t) \int_{-\infty}^{+\infty} \tilde{z}(\tau) g_o^*(\tau - t) \exp(-j\omega\tau) d\tau \]

\[ \tilde{z}(\tau) = z(\tau) \Phi_c^R(\tau) \Phi_{c,t}^M(\tau) \]

\[ \Phi_c^R(\tau) = \exp(-jc\tau^2/2) \]

\[ \Phi_{t,c}^S(\tau) = \exp(jc\cdot\tau) \]

\[ \omega_0 \]

Frequency

IF(t)

σ

c

\[ \theta \]

\[ \sigma' - \sigma \]

\[ \sigma ' - c \]

\[ c't \]

\[ \Delta IF_i(t; \sigma) \]

\[ \omega_0 \]

Time

Frequency

IF(t)

σ

IF_i(t)

\[ \kappa_p(t_0) \]

IF_i(τ) - κ_p(τ)

\[ t_0 \]

Time
Problem: Parametric TFAs lack of general theoretical framework to implement

Contribution: General parametric TF transform and dual definition

\[
TF_S(t_0, \omega; P) = \int_{-\infty}^{+\infty} \tilde{z}(\tau) g^*_{s} (\tau - t_0) \exp(-j \omega \tau) d\tau
\]

\[
\begin{align*}
\tilde{z}(\tau) &= z(\tau) \Phi^R_P(\tau) \Phi^S_{t_0,P}(\tau) \\
\Phi^R_P(\tau) &= \exp\left[-j \int \kappa_P(\tau) d\tau\right] \\
\Phi^S_{t_0,P}(\tau) &= \exp[j \tau \cdot \kappa_P(t_0)]
\end{align*}
\]

Problem: Traditional TFAs need to balance the trade-offs between concentration and cross-term and between time and frequency resolution.

Contribution: New parametric TFAs are constructed using different kernels and corresponding kernel parameter estimators are developed.

Parameter estimation

TFD dependent estimator (TF domain)  Model-based estimator (parameter space)

\[ FCI_p = \sum_{\omega} G_s(\omega; P)^4 \]

\[ G_s(\omega; P) = \int_{-\infty}^{+\infty} Rz(\tau) \exp(-j\omega\tau) d\tau \]

\[ = \int_{-\infty}^{+\infty} z(\tau) \phi_P^R(\tau) \exp(-j\omega\tau) d\tau \]

\[ = \int_{-\infty}^{+\infty} z(\tau) \exp \left\{-j \left[ \int \kappa_P(\tau) d\tau + \omega \tau \right] \right\} d\tau \]
Multi-component signal

How to analyze multi-component signals?

\[ s(t) = \sin(60\pi t + 12\pi \sin(\pi t / 6)) + \sin(0.7\pi t^2 + 25\pi t + 25) \]
**Problem:**
Traditional TFA suffers from the poor concentration or interference of cross-term in the case of multi-component signals.

**Contribution:**
TFR fusion and signal decomposition were developed for multi-component signals.

Multi-component signal

Image processing

- High-pass 2D filter
- Connectivity labeling
- Rotate and filter

Rotate

Filter and recover
Application 1 - Hydraulic turbine

Shut-down stage
Continuous and non-stationary process
Rich information
Multiple non-stationary components

Theory and methods
- Parametric TFA
- Kernel construction and estimation
- Multi-component
  - TFR fusion
  - Signal decomposition

Applications
- Vibration analysis of rotatory machines with variable speed
- Dispersion analysis of guided waves
- Nonlinear system identification
Problem:
Traditional TFA cannot characterize time-frequency pattern of multi-component non-stationary signal accurately

Contribution:
The proposed multi-component signal analysis method provides better solution of analyzing such signal in TF domain

Application 2 - Lamb wave analysis

Dispersion curves

Structure health detection

- Nd:Yad Pulse laser 532nm
- Specimen
- Digital oscilloscope 2.5GS/s, 100MHz
- Laser Doppler Velocimeter 0-2MHz
- Frequency conversion unit
- Trigger
- Mirror
- Biconvex lens

Theory and methods
- Parametric TFA
- Kernel construction and estimation

Multi-component
- TFR fusion
- Signal decomposition

Applications
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- Nonlinear system identification

Dispersion curves

S0, S1, S2, A0, A1, A2, A3
Application 2: Guided wave analysis

TFR manipulation based on parametric TFA of frequency domain

Simulated Lamb wave signal

Nonlinear (time-varying) system
1. varying restoring forces
2. varying natural frequencies
### Application 3 - System Identification

1. Extract TF features using parametric TFA (IF & IA);
2. Estimate mode shape;
3. Reconstruct backbone;
4. Estimate nonlinear stiffness and coupling coef.

\[
\ddot{\phi} + 0.05\dot{\phi} + \phi + \phi^3 - 0.8\xi - \xi^3 = 0
\]
\[
\ddot{\xi} + 0.05\dot{\xi} + 5.4\xi + 0.5\xi^3 - 0.5\phi - 0.5\phi^3 = 0
\]

<table>
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<th>STFT</th>
<th>PCT</th>
<th>Real value</th>
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<tr>
<td>(\alpha_1)</td>
<td>1.3638</td>
<td>0.9963</td>
<td>1</td>
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<tr>
<td>(\alpha_3)</td>
<td>1.4856</td>
<td>1.0057</td>
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<td>(\gamma_2)</td>
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<td>(\lambda_3)</td>
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<td>0.5</td>
</tr>
</tbody>
</table>

**Backbone and coupling**

**TFR of \(\phi\)**

**TFR of \(\xi\)**
Publications


Thanks!