TWO-DIMENSIONAL ENTROPY METHOD BASED ON GENETIC ALGORITHM

WANG Lei, SHEN Ting-zhi

(Dept. of Electronic Engineering, Beijing Institute of Technology, Beijing, 100081, China)

Abstract—Image segmentation represents the first step in many applications in image understanding. Thresholding is a simple and important in image segmentation. One-dimensional entropy does not take into account the spatial correlation between the pixels in an image. Two-dimensional entropy uses both the gray value of an pixel and the local average gray value of it, and thus provides better results. But for more accurate thresholding one has to pay the price of time. The proposed method introduces Genetic Algorithm into two-dimensional entropy method to overcome its drawback of computationally expensive. Experimental results show that the proposed method can save computational time when it provides good quality segmentation.

Key words—image segmentation; Genetic Algorithm; entropic method; fitness function

Segmentation is a process of partitioning the image into some non-intersecting regions such that each region is homogeneous and the union of no two adjacent regions is homogeneous[1]. Segmentation is the first essential and important step of any automated image understanding process, and also remains an old and difficult problem. Thresholding of the gray-level values is a popular tool used in image segmentation. Since the segmentation quality relies much on the threshold, it is crucial to choose a proper threshold.

Let \((x, y)\) be the spatial coordinate of the image, and \( G = \{0,1,2,\cdots,L-1\} \) be a set of positive integers representing gray levels (by convention, the gray level 0 is the darkest and the gray level \( L-1 \) is the lightest). The brightness (i.e., gray level) of a pixel with coordinate \((x, y)\) is denoted as \( f(x, y) \).

Let \( t \in G \) be a threshold and \( B = \{b_0, b_1\} \) be a pair of binary gray level and \( b_0, b_1 \in G \). The result of thresholding an image function \( f(\cdot, \cdot) \) at gray level \( t \) is a binary image function \( f_t \), such that

\[
f_t(x, y) = \begin{cases} b_0 & f(x, y) < t \\ b_1 & f(x, y) \geq t \end{cases}
\]

In general, a thresholding method is one that determines the optical value \( t^* \) based on a certain criterion. If \( t^* \) is determined solely from the gray level of each pixel (e.g., the one-dimension histogram), the thresholding method is point-dependent. If \( t^* \) is determined from the local property in the neighborhood of each pixel (e.g., the two-dimension histogram or co-occurrence matrix), the thresholding method is region-dependent.

Let the number of pixels with gray level \( i \) be \( m_i \). Then the total number of pixels in a given image is

\[
M = \sum_{i=0}^{L-1} m_i ,
\]

The probability of occurrence of gray level \( i \) is defined as
\[ p_i = \frac{m_i}{M}. \]

1. One-dimensional entropic method

The first entropy based principle was proposed by Pun\(^2\)[3]. In the entropy method, a criterion function by applying information theory is defined to maximize the information measure between two classes (object and background). The criterion selects a priori probability distributions when very little or no information is known.

Let \( t \) be the threshold, and class entropies are defined as

\[
H(O) = H_t, \quad H(B) = H_{L-1} - H_t,
\]

where

\[
H_t = -\sum_{i=0}^{t} (p_i \ln p_i), \quad H_{L-1} = -\sum_{i=0}^{L-1} (p_i \ln p_i).
\]

By correcting some mathematical errors in the method of Pun, Kapur et al. proposed a new maximum entropy principle for image thresholding\(^4\). Method of Kapur et al. is considered superior to other entropy thresholding algorithms\(^5\) which is employed in this paper.

Let \( t \) be the threshold, and probabilities corresponding to the object and background are

\[
O: \frac{P_0}{P_t}, \frac{p_t}{P_t}, \ldots, \frac{p_t}{P_t},
\]

\[
B: \frac{P_{t+1}}{(1-P_t)}, \frac{P_{t+2}}{(1-P_t)}, \ldots, \frac{P_{L-1}}{(1-P_t)}.
\]

Class entropies are thus defined as:

\[
H(O) = \ln P_t + \frac{H}{P_t},
\]

\[
H(B) = \ln(1-P_t) + H_{L-1} - H_{t}/(1-P_t),
\]

Where

\[
P_t = \sum_{i=0}^{t} p_i, \quad H_t = -\sum_{i=0}^{t} (p_i \ln p_i), \quad H_{L-1} = -\sum_{i=0}^{L-1} (p_i \ln p_i).
\]

The information between the two classes proposed by Kapur et al. is defined and rewritten as a function \( \psi(t) \):

\[
\psi(t) = H(O) + H(B) = \ln P_t(1-P_t) + \frac{H}{P_t} + H_{L-1} - H_{t}/(1-P_t).
\]

The optimal threshold \( t^* \) is the one at which the function \( \psi(t) \) is maximized:

\[
t^* = \underset{t \in G}{\text{ArgMax}} \psi(t).
\]

2. Two-dimensional entropic method

In reference \(^4\), the authors raised the following concern in the conclusion of their paper: “What happens if two different pictures have the same gray-level histogram and thus the same threshold? Will it be suitable for both?” They also suggested that a second-order static or some local property with the entropic concept of
thresholding might give a better insight into these problems.

Abutaleb [6] extends the method of Kapur et al. using two-dimensional entropies. The two-dimensional histogram entropies are obtained from the two-dimensional histogram that is determined by using the gray value of the pixel and the local average gray value of the pixel.

Let the average gray-level values of each pixel’s neighborhood are from 0 to $L - 1$, as well as the gray level values of each pixel. At each pixel, both the average gray-level value of the neighborhood and its gray level are calculated. This forms a pair that belongs to a 2-dimensional bin: the pixel gray level and the average gray level of the neighborhood. Let $f_{ij}$ be the total number of occurrence (frequency) of a pair $(i, j)$, and the joint probability mass function (PMF) $p_{ij}$ is defined as

$$p_{ij} = \frac{f_{ij}}{M}, \quad i, j = 0, 1, ..., L - 1.$$  

If the threshold is located at the pair $(s, t)$, then the total area under $p_{ij}$ ($i = 1, 2, ..., s$ and $j = 1, 2, ..., t$) must equal one. Thus, a normalization process is needed and this results in a modified entropy for object $O$, $H(O)$,

$$H(O) = -\sum_{i=0}^{s} \sum_{j=0}^{t} \frac{p_{ij}}{P_{st}} \ln \frac{p_{ij}}{P_{st}} = -\frac{1}{P_{st}} \sum_{i=0}^{s} \sum_{j=0}^{t} \left(p_{ij} \ln p_{ij} - p_{ij} \ln P_{st}\right) = \ln P_{st} + \frac{H_{st}}{P_{st}},$$

where

$$P_{st} = \sum_{i=0}^{s} \sum_{j=0}^{t} p_{ij}, \quad H_{st} = -\sum_{i=0}^{s} \sum_{j=0}^{t} p_{ij} \ln p_{ij}.$$  

In a similar manner, an entropy for background $B$, $H(B)$, is defined as

$$H(B) = -\sum_{i=s+1}^{L-1} \sum_{j=t+1}^{L-1} \frac{p_{ij}}{(1 - P_{st})} \ln \frac{p_{ij}}{(1 - P_{st})} = -\frac{1}{1 - P_{st}} \sum_{i=s+1}^{L-1} \sum_{j=t+1}^{L-1} \left[p_{ij} \ln p_{ij} - p_{ij} \ln (1 - P_{st})\right]$$

$$= \ln(1 - P_{st}) + \left(H_{L-1-L-1} - H_{st}\right) / (1 - P_{st}).$$

The expression for $H(B)$ is valid as long as $p_{ij} \approx 0$ in the two regions defined by $(i = s + 1, ..., L - 1$ and $j = 1, 2, ..., t)$ and by $(i = 1, 2, ..., s$ and $j = t + 1, ..., L - 1)$ . This assumption is reasonable since in many situations the off-diagonal probabilities are of negligible value. The assumption made will reduce the computation time.

The entropy-based function $\psi(s, t)$ is defined as the sum of the two entropies, viz.,
The algorithm then searches for the values of \((s,t)\) that maximizes \(\psi(s,t)\), viz.,

\[
t^* = \underset{s,t \in G}{\text{ArgMax}} \psi(s,t).
\]

3. Genetic Algorithm

Genetic Algorithm is created by analogy with the processes in the reproduction of biological organisms\[7\]. It can be classified as guided random search evolution algorithms that uses probability to guide its search.

Simple Genetic Algorithm (SGA) employs the following genetic operators in a search procedure: proportional reproduction, one-point crossover, and simple mutation. It maintains a constant-sized “population” of candidate solutions, known as individual. The initial seed population from which the genetic process begins can be chosen randomly or on the basis of heuristics, if available for a given application. At each iteration, known as a “generation”, each individual is evaluated and recombined with others on the basis of its overall quality of “fitness”. In this paper, we improve the proportional reproduction by applying Remainder Stochastic Sampling with Replacement (RSSR). Two-point crossover is employed to replace one-point crossover. The elitist strategy is also employed to assure the Genetic Algorithm’s convergence.

3.1 Improvement to basic reproduction operator

The proportional reproduction operator in SGA decides the expected number of times an individual is selected for recombination, which is proportional to its fitness relative to the rest of the population.

Let \(N\) be the population size, \(\text{fit}_i\) be the \(i\)th individual’s fitness, the possibility for this \(i\)th individual is defined as

\[
p_i = \frac{\text{fit}_i}{\sum_{i=1}^{N} \text{fit}_i} \quad (i = 1,2,\cdots,N).
\]

An obvious drawback of “proportional reproduction” is that individuals with high fitness may not be selected and reproduced especially when the generation turns to be relatively small. The selection error is caused by the random selection techniques.

Remainder Stochastic Sampling with Replacement (RSSR)\[9\] is employed in the paper to improve the basic proportional reproduction operator. RSSR reproduction operates as follow steps:

1) Let \(N_i\) be the expected number of times an individual is selected for recombination, which may possibly not be an integer.

\[
N_i = \frac{N \cdot \text{fit}_i}{\sum_{i=1}^{N} \text{fit}_i} \quad (i = 1,2,\cdots,N).
\]

2) Let \(\lfloor N_i \rfloor\) be the integer part of \(N_i\) so as to construct \(\sum_{i=1}^{N} \lfloor N_i \rfloor\) individuals of the next offspring.
3) Replace each individual’s fitness by its new fitness which is calculated as
\[ \text{fit}_i = \frac{\sum_{i=1}^{N} \text{fit}_i}{N}. \]

Then, get the left \( N - \sum_{i=1}^{N} \lfloor \text{fit}_i \rfloor \) individuals for the next offspring by applying the simple proportional reproduction.

By combining random selection techniques and certain selection techniques, RSSR reproduction assures individuals with higher fitness than average fitness being selected for recombination, thus turns to have less selection error.

3.2 Two-point crossover and simple mutation

One-point crossover employed in SGA chooses a point at random, called the crossover point, and exchanges the segments to the right of this point. It has one major drawback in that certain combinations of schema cannot be combined in some situations.

Two-point crossover chooses two points at random, and exchanges the segments between them. Research indicates that two-point crossover performs better than one-point crossover[10].

Simple mutation\[^8\] is related to a bit (gene) that could be changed according to the mutation probability.

3.3 The elitist strategy\[^8\]

Although the RSSR reproduction can protect the best individual of each generation, it is still possible for the best individual to be disrupted by the crossover and mutation. Thus it is necessary to introduce the elitist strategy by which the best individual is copied into the succeeding generation directly.

4. Image segmentation based on Genetic Algorithm using two-dimensional entropy

There are two major considerations in practical application of GA: one is how to map the optimization problem into GA’s search space, i.e., how to encode the individuals so as to solve the optimization problem. The other one is how to choose proper fitness function.

The image segmentation deals with pixels of different gray-level. Since the practical image used in this paper is 128-level, each gray level in the population is represented by 7 bits. Since 2-dimensional histogram contain both the pixel gray level and the average gray level of the neighborhood, the chromosomes are encoded as 14 bit strings, with the first 7 bits representing the pixel gray level and the second 7 bits representing the average gray level of the neighborhood.

The fitness function is derived from two-dimensional entroopic method, which is defined as
\[ \psi(s,t) = H(O) + H(B) = \ln \frac{P_{st}}{P_{st}} (1 - P_{st}) + \frac{H_{st}}{P_{st}} + \left( \frac{H_{L-1L-1} - H_{st}}{1 - P_{st}} \right). \]

In this approach, the population size is 10, the crossover probability is 0.8, the mutation probability is 0.01, predetermined number of generations is 40.

An abstract procedure of the Improved Adaptive Genetic Algorithms is given below, where \( N(k) \) is a population of candidate solutions to a given problem at generation \( k \):

1. \( k = 0 \), generate an initial population \( N(k) \).
2. Compute the population \( N(k) \).
3. Perform the GA's reproduction, crossover, and mutation.
4. Finish the GA and get the final optimal threshold if the predetermined number of generations is reached or
the optimal threshold of each generation remains same for 10 generations, else return to (2).

5. Experiments and results

(a) original image         (b) one-dimensional entropic method

(c) two-dimensional entropic method                     (d) the proposed method

FIG. 1. Gray-level image of “bear”

TABLE 1. Comparison of different parameters for “bear” image

<table>
<thead>
<tr>
<th></th>
<th>optimal threshold</th>
<th>computational time/s</th>
</tr>
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<tbody>
<tr>
<td>one-dimensional entropic method</td>
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</tr>
<tr>
<td>two-dimensional entropic method</td>
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<td>24.595</td>
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<tr>
<td>the proposed method</td>
<td>29</td>
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6. Conclusion

First, the one-dimension entropy method is one of those methods that do not take into account the spatial correlation between the pixels in an image. The two-dimension entropy method uses both the spatial gray-level distribution and the gray-level distribution. Thus makes a more thorough use of the original image and yields better results.

Second, Abutaleh indicates that when compared to the one-dimension entropy, the two-dimension takes more computational time (an order of magnitude increase). In this paper, the Genetic Algorithm is applied to segmentation, which is considered a fast evolution algorithm itself. At the same time, we have improved the Simple Genetic Algorithm by using better reproduction and crossover operators. Thus the proposed method makes up the two-dimension entropy method’s drawback of being time consuming, and yields satisfactory segmentation results.

Finally, although the proposed algorithm can save considerable computational time and assure segmentation time compared with two-dimension entropic method, it still takes more computational time than one-dimension
entropic method. We still have to pay for the increase in accuracy by spending more computational time.

REFERENCES

基于遗传算法的二维直方图熵法
王蕾 沈庭芝
（北京理工大学电子工程系，北京，100081）

摘要：图像分割方法中的二维直方图熵法的计算时间较长。为了提高其计算效率，本文将遗传算法和二维直方图熵法结合起来，同时考虑了孤立象素点的灰度信息和象素点的空间相关性，并对简单遗传算法的复制和交叉算子进行了改进，利用遗传算法高效快速的特点，克服了二维直方图熵法的缺点。实验结果表明，本文算法在保证分割质量的同时，提高了运算效率。

关键词：阈值法；图像分割；直方图熵法；遗传算法

Abstract—Two-dimensional entropic method has to pay the price of time when applied to image segmentation. In this paper, the Genetic Algorithm is introduced to improve the two-dimensional entropy method’s computational efficiency. The proposed method uses both the gray value of a pixel and the local average gray value of an image. At the same time, the Simple Genetic Algorithm is improved by using better reproduction and crossover operators. Thus the proposed method makes up the two-dimension entropy method’s drawback of being time consuming, and yields satisfactory segmentation results. Experimental results show that the proposed method can save computational time when it provides good quality segmentation.

Key words—Thresholding; image segmentation; entropic method; Genetic Algorithm
FIG. 1. Gray-level image of “bear”

(a) original image
(b) one-dimensional entropic method
(c) two-dimensional entropic method
(d) the proposed method