Abstract—In this paper we combine Gaussian process regression and impedance control, to illicit robust, anthropomorphic, adaptive control of a powered ankle prosthesis. We learn the non-linear manifolds which guide how locomotion variables temporally evolve, and regress that surface over a velocity range to create a manifold. The joint set of manifolds, as well as the temporal evolution of the gait-cycle duration is what we term a locomotion envelope.

Current powered prostheses have problems adapting across speeds. It is likely that humans rely upon a control strategy which is adaptable, can become more robust and accurate with more data and provides a nonparametric approach which allows the strategy to grow with the number of observations. We demonstrate such a strategy in this study and successfully simulate locomotion well beyond our training data. The method we propose is based on common physical features observed in numerous human subjects walking at different speeds. Based on the derived locomotion envelopes we show that ankle power increases monotonically with speed among all subjects. We demonstrate our methods in simulation and human experiments, on a powered ankle foot prosthesis to demonstrate the effectiveness of the method.

I. INTRODUCTION

The field of rehabilitation and bipedal walking is currently undergoing a minor revolution [1], [2], due to recent advancements in robotics, control theory and machine learning. Technologies are being developed which actively aid, or can even restore legged locomotion. This is not just restricted to amputees, but also those suffering muscle impairments. These advances in powered prostheses, promise to improve the quality of life for those who have severely reduced mobility. The United States alone is home to almost two million amputees, and amongst these there are six hundred thousand people with below knee amputation, who are in need of suitable ankle-foot prostheses, to regain an agile and stable gait. There are a further 4.7 million stroke survivors, with another 700K added each year, many of whom could benefit from a powered ankle-foot orthosis [3] to improve their ambulatory performance.

Current available solutions are mostly unsatisfactory and consist mainly of passive prostheses. These provide a spring-like behaviour and lends the user slow walking performance, compared to the self-selected walking speeds of a healthy subject. Hence, lots of people could benefit from a powered ankle-foot prosthesis [4] using universal function approximation methods, combined with impedance control. The adaptive property helps the user to change the walking speed, and the nonparametric property, allow the control scheme higher precision with more data. The proposed approach is general and can be applied to any ankle-foot prostheses. Hence, lots of people could benefit from a powered ankle-foot prosthesis [3] to improve their ambulatory performance.

Fig. 1: Schematic diagram of the ankle-foot prosthesis with five degrees of freedom, as well as physical prototype. In (a) \( T_x \) and \( T_y \) are translations along the the x and the y axis. \( R_z \) is rotation about the z axis. \( F_1 \) and \( F_2 \) are ground forces.

1) We use Gaussian process (GP) regression to learn a multivariate function describing the power required at the ankle joint as a function of speed and position in the gait-cycle.
2) We combine impedance control with the multivariate functions which we term a locomotion envelope.
3) We demonstrate our methods both on simulation and hardware experiments.

Initially we consider the sagittal degree of freedom (DoF).

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These results are applicable to orthoses as well, but that is outside the scope of this paper.
The shin is modelled with a rigid body connected to a planar floating base to simulate walking using recorded human walking data which is shown in Fig. 1. The ankle-foot prosthesis and its interactions with the environment are modelled in the Robotran simulator [5] as shown in §V.

II. RELATED WORK

Lenzi et al. [6] note that “available controllers for powered transfemoral prostheses cannot generalise across different walking speeds”. Where the most common solution used for control of active prostheses is to match the torque-angle profile of a healthy human ankle at the powered prosthetic joint [7], [8], [9]. In previous work [10] the authors describe a finite-state impedance control approach to control a prosthesis during walking and standing. This control method often leads to a few fixed walking speeds but lacks the adaptability required to seamlessly change the velocity or step size following the user’s intention. In addition, gathering human experimental data at different speeds is limited to a few discrete values and it is difficult to perform numerous experiments to derive walking data at intermediate walking speeds or step sizes [11], [12], [13].

In order to address the adaptation problem of this control method, the authors [14] proposed a reflexive neuromuscular model with positive force feedback. They showed that the proposed method can adapt to changes in the walking speed and floor inclination. Further, in [15] the authors proposed a neuromuscular reflexive model with speed adaptation for a powered ankle-foot prosthesis and tested it at three walking speeds: 0.75, 1.0, and 1.25m/s. Another study [16] extracted electromyography (EMG) signals from a patient, who had undergone targeted muscle reinnervation surgery, and then used these to provide a robust control mechanism for an ankle and knee prosthesis to walk with a fixed speed on level ground, stairs, and ramps with a 10° slope. The same group of researchers [17] demonstrate a control approach for a transfemoral prosthesis which regulates the ankle and knee joint torque by estimating the walking phase and speed. A two dimensional lookup table with a low-pass filter was used to encode torque-angle curves for two walking speeds. In [18], [19] the authors proposed a virtual constraints based control scheme, which used a human-inspired phase variable to adapt to speed variations.

The authors in [20] suggest that active powered prosthesis control falls into four categories: echo control, finite state impedance control, EMG based control and central pattern generator (CPG) based control. Conversely, Lenzi et al. [6] suggest that effective speed adaptation has been successful using two approaches. The first, described in [21], proposes a method that mimics human muscle reflexes. This allows the prosthesis to adopt velocity adaptation by virtue of changing the torque output without actually “measuring the walking or cadence” [6]. Although demonstrating impressive results, there are some drawbacks. Specifically, because their guiding metric is the metabolic cost of transport, for five different velocities, across which they regress, it is difficult to ascertain how well their control schema transitions between velocities. Secondly, Lenzi et al. [6] describe a taxonomy which uses preprogrammed ankle-torque profiles. In order to cope with velocity variations, Holgate et al. [3] modulated the ankle trajectory in time and amplitude, allowing their subject to transition between velocities. But this is a parametric method, relying on look-up tables for fitting and has no uncertainty bounds. Finally, in [6] they propose a method which imitates the basic velocity adaptation mechanism used by healthy (i.e. intact) legs. They employ quasi-stiffness profiles (we show diagrams of these in our result section too) of an intact leg, which they directly encode into their controller, and then interpolate between them based on their intention estimation. Theirs is most likely the work that comes closest to ours, but whilst they use a PD controller, we use an impedance controller, and probabilistic interpolation and extrapolation, which also gives us uncertainty bounds on our predictions. Further reviews of control strategies for lower extremity prostheses can be found in [22].

Like us, GPs have been used before for similar purposes. In [23] they use the GP dynamics model, a dimensionality reduction method, to create a low-dimensional representation of walking motion, extracted from 50 subjects. They do this for three different speeds. This is not a control scheme, but a rehabilitation method, which can generalise in between the training speeds (their reconstruction errors are high though: > 10%) and is used for gait training of subjects with hemiplegia. In [24] they instead make use of standard GP regression (GPR) to optimise gait for quadruped and biped robots. Though they do not deal specifically with velocity adaptation and regulation (rather environment adaptation) their ideas are relevant to our discussion, as environmental adaptation is the next logical step for our method. Similarly [25] used GPR to generate a model for gait pattern prediction for one speed (3km/h). Though not what we propose, it does suggest a validation for our method, since the paradigm remains the same; theirs is a prediction in space, ours in space and time.

This paper is organised as follows; we present locomotion envelopes in §III followed by §IV where we provide some insight and analysis of experimental human walking data. In §V we provide an exposition of the simulation framework. The simulation results are presented in §VI. The experimental validations are provided in §VII followed by the conclusion & future work in §VIII.

III. LOCOMOTION ENVELOPES

The term locomotion envelope (LE) refers to the joint set of multivariate regression surfaces which, appropriately applied, confers upon the user the ability to synthesise natural and robust bipedal locomotion (that same envelope also contains learned temporal regression functions, guiding the evolution of stride duration, across a sought range of gait speeds). We seek to regress physical properties from very sparse observational data, to ascertain the power required.

²We derive the term from aerodynamics in which the flight envelope, service envelope or performance envelope of an aircraft refers to the capabilities of a design in terms of airspeed and load factor or altitude.
at the ankle-joint, when walking at different velocities (see §IV). This can be viewed as a learning problem, where we are interested in learning the multivariate regression manifolds of the aforementioned properties.

There are a multitude of options which one could use to construct these envelopes, we posit however that the most suitable is GPR. There are a number of reasons for this, some of which we expand upon here (for a detailed discussion see [26, §1.1.3]). First, given observations and a kernel, the posterior predictive distribution can be found exactly in closed form [26]. Secondly, by nature of its construction, expressivity is considerable, allowing us to incorporate a host of modelling assumptions and domain knowledge. Finally, as noted by [26]; given a fixed kernel, the GP posterior allows us to integrate exactly over hypotheses, hence overfitting is less of an issue than in comparable model classes. A brief exposition of the methodology follows in the next section.

A. Gaussian processes background

A Gaussian processes [3] (GP) is a method for universal function approximation i.e. some realisation of a GP, with some kernel, is arbitrarily close to the function under study, to within some norm. Thus we can approach the multivariate function learning problem by placing a prior distribution on the regression function using a GP [27]. With a GP we can define a distribution over functions \( f(x) \sim \mathcal{GP}(m(x), k(x, x')) \), parametrised in terms of a mean function \( m(x) \) and a covariance function \( k(x, x') \). A GP is fully specified by these two functions. Throughout we assume we have a dataset \( D = \{(x_i, y_i) \mid i = 1, \ldots, n\} \) of \( n \) input predictor vectors, aggregated in design matrix \( X \) of size \( n \times D \) s.t. \( x_i \in \mathbb{R}^D \). With corresponding target values \( y_i \in \mathbb{R}^1 \), aggregated in a target vector \( y = [y(x_1), \ldots, y(x_n)]^T \).

If we let \( f(X) = [f(x_1), \ldots, f(x_n)]^T \) then GPs provide a flexible class of models in which any finite-dimensional marginal distribution is Gaussian, which is to say if

\[
f(X) \sim \mathcal{GP}(m(X), k(X, X)) \quad \text{then} \quad \mathbb{P}(f(x_1), f(x_2), \ldots, f(x_n)) = \mathcal{N} \left( \begin{bmatrix} m(x_1) \\ \vdots \\ m(x_n) \end{bmatrix}, \begin{bmatrix} k(x_1, x_1) & \cdots & k(x_1, x_n) \\ \vdots & \ddots & \vdots \\ k(x_n, x_1) & \cdots & k(x_n, x_n) \end{bmatrix} \right). \tag{2}
\]

The \( n \times 1 \) sized mean vector \( m_i = m(x_i) \) is specified by a mean function \( m(x) = \mathbb{E}[f(x)] \), which represents an initial guess at the regression function \( f(\cdot) \). The covariance function \( k(x, x') = \text{cov}(f(x), f(x')) \) specifies the covariance between the process at any two points, resulting in a \( n \times n \) covariance matrix \( K_{ij} = k_{\theta}(x_i, x_j) = k(x_i, x_j) \). The covariance function and its hyperparameters \( \theta \), control the smoothness of realisations from the GP and the degree of shrinkage towards the mean [28]. Different covariance functions can be used to add structural prior assumptions like smoothness, non-stationarity, periodicity and multi-scale or hierarchical structures [28]. Sums and products of Gaussian processes are also GPs which allows easy combination of different covariance functions.

1) Regression: Ultimately our goal is to infer target values at \( n_\ast \) test points not in the design matrix \( X \). Given a Gaussian observation model, with additive noise [28], [29] modelled by a GP: \( y(x) \mid f(x) \sim \mathcal{N}(f(x), \sigma^2) \); its predictive distribution [27, §2.2] over test points \( X_\ast \) has the form

\[
\mathbb{P}(f(X_\ast) \mid X, y, X_\ast, \theta, \sigma^2) = \mathcal{N}(\tilde{f}_\ast, \text{cov}(f_\ast)) \tag{3}
\]

where

\[
\tilde{f}_\ast = m(X_\ast) + K(X, X_\ast) \cdot \Omega \cdot y \tag{4}
\]

\[
\text{cov}(f_\ast) = K(X_\ast, X_\ast) - K(X, X_\ast) \cdot \Omega \cdot K(X, X_\ast) \tag{5}
\]

where \( \Omega \triangleq [K(X, X) + \sigma^2 I]^{-1} \) and all covariance matrices implicitly depend on the kernel hyperparameters \( \theta \) [29, §2.1]. Typically we consider GPs with a zero mean function. This is not a limitation. Since the mean of the posterior process is not confined to be zero. But there are many reasons why one may want to explicitly model the mean function, including interpretability of the model and convenience of expressing prior information [27, §2.7]. An example of GP regression is shown in fig. [2].

2) Model selection: Usually we do not a priori know the values of the hyperparameters, but might know broadly where they should feature in order to return a model of high accuracy. A more structured approach to model learning is to optimise the marginal likelihood (ML) [28, §21] of the data w.r.t. the hyperparameters \( \theta \), by invoking kernel learning. We can analytically marginalise the GP to obtain the ML [28] of the targets conditioned only on the covariance hyperparameters

\[
\log \mathbb{P}(y \mid \theta) = -\frac{n}{2} \log(2\pi) - \frac{1}{2} \mathbb{E}[f(X, X \mid \theta)]
\]

where \( K_\theta \) is short for \( K(X, X \mid \theta) \). Where we seek the partial derivatives of eq. [4] w.r.t. \( \theta \), which can be found analytically [27]. Wilson et al. [29, §2.1] explain that eq. [4] separates automatically into calibrated model fit and complexity terms which are optimised to learn the kernel hyperparameters \( \theta \). This fundamental property of GPs, make them particularly powerful since it allows one to balance the capacity of the model and suitability (fit) to the observations [26, §1.1.1], thus yielding a measure of the model uncertainty.

B. Gait cycle stride time regression

Each envelope needs to scale each manifold by time as the prosthesis accelerates or decelerates. This is a non-linear relationship, which, like the manifolds, also needs to be approximated to find the one-dimensional regression function. To aid our exposition and comparison, we employ the study by [12], wherein motion capture data was collected for five subjects walking and running on a force-plate instrumented treadmill. The subjects walked at 1.00, 1.25, 1.50 and 1.75m/s and ran at 2.0, 3.0, 4.0 and 5.0m/s. The violin

\[ \text{For a comprehensive list of notation used within, see the preface of [27].} \]
Fig. 2: Workflow used in obtaining an ankle plantarflexion angle regression manifold, for subject 17, from the Moore dataset [13]. Figure 2a shows the training data used, fig. 2b shows a heatmap the predictive mean at the test points and fig. 2c a three dimensional visualisation of the predictive mean at the test points. Refer to fig. 1 for the reference frame.

IV. ANALYSIS OF HUMAN AMBULATION

In this section, we verify the common physical feature observed in all healthy subjects as they walk faster. We show that the power at ankle increases monotonically with speed. The power is computed by deriving the product of ankle moment and ankle angular velocity and then normalized as shown in fig. 4 for two subjects. This results confirms the hypothesis and one of the contributions of this paper that an increase in walking speed occurs as a result of increase in the power exerted by humans at the ankle. Human data suggests that this feature is unique to the ankle joint where a clear increase in power at push-off can be observed. This phenomenon is not present when looking at the knee or the hip joints in sagittal plane during human walking. This is a universal feature that can be observed in human data and reproduced in powered ankle-foot prostheses. All that is required to realise this mechanism is sensory information (such as IMUs) which informs the control system regarding the speed of walking.

The plots shown in fig. 4 also clarify when the push-off phase starts and when it ends and the ankle enters the swing phase. In the swing phase the power at the ankle drops to a value close to zero, as expected. Although the power plots for other subjects follow the same trend they are not included for brevity. Further, the experimental data used for this study, comes from the set of experiments conducted by [13]. Therein the authors collected a rich gait dataset with the help of fifteen subjects, walking at three speeds on an instrumented treadmill. Each trial consisted of 120s of normal walking and 480s of walking while being longitudinally perturbed during each stance phase with pseudo-random fluctuations in the speed of the treadmill belt. Although, we are primarily interested in the normal walking observations, but note that the methods developed...
herein would also form an interesting study if applied to the perturbed data as well. The dataset contains: full body marker trajectories and ground reaction loads. All of which was collected at 0.8, 1.2 and 1.6m/s, for each subject.

The next section presents the results of applying the locomotion envelopes on a mathematical model of the ankle-foot prosthesis.

V. SIMULATION SETUP

A planar model of the ankle-foot prosthesis is developed in Robotran [5]. Figure 1 shows four revolute joints in total where three joints \( q_1, q_2, \) and \( q_3 \) are the floating base joints specifying the \((x, y)\) position and orientation of the shin in the plane. The fourth joint, \( q_4 \) represents the planar pitch motion of the ankle that is actively controlled during walking. The fifth joint \( q_5 \) is a passive toe with spring stiffness of 90Nm/rad and damping of 3Nm·s/rad that was added to better accommodate human foot kinematics. We let \( q \triangleq [q_1, q_2, q_3, q_4, q_5] \). The equation of motion for this system is

\[
M(q)\ddot{q} + c(q, \dot{q}) = \tau + J^T F_{ext} \tag{7}
\]

where \( M \) is the mass inertia matrix, \( c \) is the vector of Coriolis, centripetal and gravity forces, \( J \) is the Jacobian matrix, and \( F_{ext} \) represents the ground reaction forces. The kinematics and dynamic parameters were extracted from each subject’s data as reported in [13]. The ground reaction forces and moment are applied at the foot’s centre of pressure. The walking simulation consists of a swing phase where the ground reactions are zero and a stance phase where a part of the foot is in contact with the ground.

In order to show the functionality of the proposed nonparametric regression methods, in changing the walking speed, inverse dynamics is performed on the inferred manifolds to illustrate acceleration and deceleration during transitions from one speed to another. These transitions can occur at any moment during the gait cycle. A virtual sensor is attached to the centre of the foot in the simulation to obtain important gait features such as step size, step frequency, velocity and acceleration. The stance phase of walking can be divided into three distinct sub-phases. The first is controlled plantarflexion (CP) that occurs right after heel strike. This phase is followed by controlled dorsiflexion (CD) that occurs when the angle between the shin and ankle starts to decrease. This is followed by powered plantarflexion or push-off phase that is the main focus of this paper and it is where energy and power is injected into the walking gait.

A. Locomotion envelopes and kernel design

In this section we apply our methodology to a set of experiments, demonstrating the utility in using locomotion envelopes for synthesising robust locomotion. First, however, we provide a brief exposition on our choice of kernels.

We use kernel-based nonparametric GPs to find a subject’s LE. Good performance for these methods is conditional on the choice of kernel structure as it encodes our assumptions about the function which we wish to learn [27, §4]. Typically this choice can be somewhat difficult, and methods have been proposed for automating the selection process [30], in our case, however, we have substantial prior information regarding the nature of our multivariate regression surfaces.

The kernel function \( k(x, x') \) determines how correlated or similar our outputs \( y \) and \( y' \) are expected to be at inputs \( x \) and \( x' \). As noted by [30]; by defining the measure of similarity between inputs, the kernel determines the pattern of inductive generalisation. In table 1 we have summarised the kernel structure for the present variables in the envelope. These are, in no particular order: ground reaction forces (GRF) \((x, y)\), ground reaction moments (GRM) \((x, y)\), marker positions \((x, y)\) as well as ankle and knee joint-angles.

<table>
<thead>
<tr>
<th>Target</th>
<th>ARD</th>
<th>Periodic</th>
<th>SE</th>
<th>RQ</th>
<th>M52</th>
</tr>
</thead>
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<tr>
<td>Joint angles</td>
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<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>GRF ((x, y))</td>
<td></td>
<td></td>
<td>✓</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Marker positions ((x, y))</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
<tr>
<td>GRM ((x, y))</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
<td></td>
</tr>
</tbody>
</table>

As we are only considering one period of the gait-cycle we use periodic kernels where beneficial to do so, yielding functions of the form \( f(x) = f(x + P) \) where the \( P \) is the period of the gait-cycle. At times the addition of a periodic kernel has no benefit to the predictive performance, in which cases it has been omitted. Further, it is clear that the function values of \( f(\cdot) \) change faster, and slower, depending on which input dimension \( x \in \mathbb{R}^{D=2} \) we are considering. Intuitively this means that directional changes is of no importance. This is too strong an assumption in our case hence why isotropic kernels are unsuitable for our application domain. Instead we employ automatic relevance determination (ARD), which appropriately scales the inputs, thus determining the ‘relevance’ of each dimension.

The final part of our kernel design is rather more crucial as it concerns the innate periodicity assumptions of our data. Whilst it is clear that gait-cycles are periodic, they are not exactly periodic. This is further reinforced by our usage of the mean gait-cycle (for one period). Thus to allow for realistic variations over time, by design we make our kernels locally periodic, by multiplying by a local kernel. This allows us to model functions that are only locally periodic, the shape of the repeating part of the function can now change over time [26]. We experimented with different local kernels each of which makes different smoothness assumptions about our data. The final form for each variable group, is shown in table 1. Where our design objective was to find natural looking, numerically consistent (with experimental data) simulations. Having considered our kernel choices, we are in a position to use them for simulation and experimental evaluation.

4For details on the Matérn kernels with \( \nu = 5/2 \) (M52), and the rational quadratic (RQ) kernel, see [27, §4.2].
VI. SIMULATION RESULTS

In this section, the locomotion trajectories are extracted from the inferred LE. These trajectories are cyclic at any speed and can accelerate or decelerate at any time during the gait cycle. These are the key properties that are highly desirable to endow upon powered ankle-foot prostheses; speed adaptation and control repeatability.

A. Accelerating and decelerating

In this simulation, a representative path on the inferred LE is chosen to simulate three speeds and their smooth transitions. The inferred joint angles and ground reaction forces and moments are used in inverse dynamics calculations, to illustrate walking with speed adaptation at three walking speeds 0.6, 1.2, and 1.8 m/s. Recall that X only contained velocities in the range $0.8 \leq v \leq 1.6$ m/s. The walking speed is measured by placing a virtual sensor on the foot in simulation that illustrates the instantaneous position, velocity and acceleration. Three gait cycles at each speed are completed, before transition. Figure 5 demonstrates accelerations using our methodology. Each speed profile is repeated over three cycles before acceleration to the next speed profile. It is interesting to note that the lowest (0.6 m/s) and the highest speed (1.8 m/s) were not available from the experimental data, and their profiles inferred by the LE.

![Fig. 5: Cartesian velocity and acceleration of the foot measured during two speed transitions, indicated by vertical dashed bars.](Image)

Having extracted a locomotion manifold, we are in a position to synthesise complex locomotion sequences, consisting of several speeds, durations, accelerations and decelerations. In fig. 6 we demonstrate several properties of our method. We are able to robustly accelerate and decelerate the simulated prosthesis, to arbitrary speeds and for arbitrary durations, for a simulated locomotion path of our choosing. The path was tracked using concatenated gait-cycles, found from the learned LE. The gait is realistic and mimics subject 6’s gait well. We demonstrate a plausible sound gait as seen in fig. 6c. In fig. 6c super-imposed curves for speeds $v = 0.6, 0.9, 1.2, 1.5$ and 1.8 m/s, to the experimental ones found at speeds 0.8, 1.2 and 1.6 m/s. The method faithfully extrapolates the curves, by carrying shape and appearance, as speed increases and decreases. In fig. 6d we conduct a small experiments on held-out data on subject 7, from [11]. We train models on $v = 0.8, 1.17, 1.64$ m/s and predict the kinetics of subject 7 at $v = 0.61$ m/s.

![Fig. 6: Simulation experiments.](Image)

As mentioned earlier, the main focus of the human experiments at this point is the push-off phase of walking and hence only the push-off part of the inferred locomotion envelopes are used to feed the impedance controller with suitable push-off trajectories based on the speed of walking. Moreover, the value of impedance during all stages of the gait cycle is kept fixed.

![Fig. 6: Simulation results.](Image)

<table>
<thead>
<tr>
<th>Method</th>
<th>RMSE [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>GPR with SE-ARD kernel</td>
<td>4.57</td>
</tr>
<tr>
<td>RBF with multiquadric kernel</td>
<td>7.30</td>
</tr>
<tr>
<td>RBF with cubic kernel</td>
<td>9.51</td>
</tr>
<tr>
<td>Nearest-neighbour interpolation</td>
<td>3.28</td>
</tr>
</tbody>
</table>

VII. HARDWARE EXPERIMENTS

Herein, we first introduce the impedance control implemented on the ankle-foot prosthesis and then present the results of an unimpaired subject walking with the ankle-foot prosthesis with the help of a knee support bracket at three different speeds. The subject provided a written consent to participate in the experiment as approved by the Michigan Technological University Institutional Review Board.

As mentioned earlier, the main focus of the human experiments at this point is the push-off phase of walking and hence only the push-off part of the inferred locomotion envelopes are used to feed the impedance controller with suitable push-off trajectories based on the speed of walking. Moreover, the value of impedance during all stages of the gait cycle is kept fixed.

![Fig. 6: Simulation results.](Image)
A. Impedance Control

Two impedance controllers are required to control each motor independently. Each impedance controller uses an external position controller with an internal torque controller. The external position controller tracks a reference trajectory (generated via LE method) and uses the angle feedback from the encoders on each motor $\theta_m$. The block diagram of the impedance controller is shown in Fig. 7. The motors actuate the ankle in dorsiflexion-plantarflexion (DP) and inversion-eversion (IE) directions using Bowden cables that form a differential drive mechanism.

The output of the external position controller is the desired torque to be generated by the motor $\tau_r$ and is the input to the internal torque controller. The internal torque controller uses torque feedback from strain gauges mounted on the foot. The strain gauges provide both the torque in dorsiflexion-plantarflexion ($\tau_{DP}$) and the torque in inversion-eversion ($\tau_{IE}$). Torques $\tau_{DP}$ and $\tau_{IE}$ are calculated from the strain gauges voltage outputs as described in previous work [4]. Due to the differential drive nature of the mechanical setup, the sum of $\tau_{DP}$ and $\tau_{IE}$ is used for the reference torque ($\tau_r$) for one of the motor controllers, and the difference of $\tau_{DP}$ and $\tau_{IE}$ is used in the other motor controller. Similarly, the reference trajectory $\theta_r$ for one of the motor controllers is the sum of the desired ankle angles in DP ($\theta_{DP}$) and in IE ($\theta_{IE}$), and for the other motor controller, the reference trajectory $\theta_r$ is the difference of the desired ankle angles in DP and in IE.

The torque controller output is the desired motor velocity $\dot{\theta}_r$ and used as input to the Maxon motor controllers [4]. The impedance controllers were implemented with a real-time frequency of 200Hz while the Maxon motor controllers used a PI controller running at 53kHz. For details see [4].

![Figure 7: Impedance controllers for prosthesis motors.](image)

B. Human walking experiments

In this section, the results of a healthy subject walking in a straight line at two different speeds with the help of the ankle-foot prosthesis are presented. In this experiment only the DP degree of freedom of the ankle-foot prosthesis is used to provide push off while IE is controlled to stay at zero. The third speed is not shown due page limitations. Figure 8 illustrate the amount of push-off trajectory and power provided by the ankle-foot prosthesis at two different speeds. It is shown that in order to sustain the balance at higher walking speeds, the increased power at the ankle-foot prosthesis is necessary. A video of this experiment is also provided with this paper to better illustrate the experimental results. Fig. 8: Experimental results of walking at two different speeds $v_1$ and $v_2$ with $R^2$ and RMSE measured for each trial. On the left hand side the walking speed is $v_1$ and the injected push off power is well below 50 Watts. On the right hand side the walking speed is $v_2$ and the injected push off power is over 100 Watts.

Our results contrasts well with one of the studies which most similar to ours, namely that by Lenzi et al. [6]. They integrate biologically accurate torque–angle curves into their controller, by encoding “a few speed-specific curves from able-bodied studies” [6], and then interpolating (but not extrapolating) between them. Nor do they have any uncertainty feedback incorporated into their controller. Nonetheless, whilst comparison is difficult (owing to the different nature of our experiments), similar RMSE and $R^2$ scores are recorded. For example, for their mid-stance to late-stance, they recorded an RMSE of 3.05° and $R^2$ of 0.7154 when subjects walked “on a treadmill at continuously varying walking speeds” [6]. That being said, because we propose a nonparametric method, the variance on our predictions will reduce as the size of $X$ grows. This is not true for comparable parametric controllers.

See the supplementary material at: [https://youtu.be/FGmN6B27RxU](https://youtu.be/FGmN6B27RxU)
In this paper, we have presented a data-driven control strategy for ankle-foot prostheses. We have demonstrated (by way of simulation) that the methodology has the capacity to allow the user to walk over a wide range of speeds, whilst also providing for fast variations outside of the training data. Where, in future work, we intend to expand upon the speed range by adopting a more domain-specific mean function for GP regression as well as employing more advanced forms of information sharing in the GP framework. Moreover, the ankle impedance was kept fixed during these experiments and future work will study variable impedance control to better accommodate natural human locomotion.

Though this work is primarily intended for the rehabilitation robotics domain, it may prove insightful to the field of bipedal humanoids. Although we have implemented the work for prostheses, the underlying theory concerns basic understanding of human locomotion, and how to adapt those insights to generalise our control strategies, to effect a singular, or desired set of locomotions. Further, we have demonstrated that taking a broadly cyclical view of human locomotion is useful since it means we can extrapolate over gait-cycles rather than full time-series data, across speeds. More importantly we have demonstrated a method for generating biologically accurate torque-angle curves, which we can repress, potentially much further than what has been shown in this work (conditioned on the discussion in the previous paragraph).

REFERENCES


