Epistemic Uncertainty in Quantum Mechanics

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Quantum mechanics (QM) is inextricably entwined with notions of uncertainty, and hence with probability \[\text{[Mermin 2004, Fuchs 2002, Ja[nes 1982],}\] however, emphasises that a distinction needs to be made in QM between its epistemic and ontologic content. Explicitly, does the uncertainty intrinsic to QM describe reality, or does it describe only our our states of knowledge about reality?

In the Bayesian \[\text{[Janes 2003]}\] paradigm, a probability is always a statement of belief about the world, never a physical model of a real process. Of course, as we are warned by \[\text{[Jeffreys 1998, Section 8.4]}\], that the symbols of probability are employed by QM does not necessarily mean that these quantities are the epistemic probabilities of Bayesian theory. A more careful examination of their origins is required.

In QM, probability emerges through the presence of density matrices, a statistical description of the state of a system under various operations. These have indeed been interpreted as purely epistemic constructs by \[\text{[Mermin 2004]}\], who quotes \[\text{[Peierls 1999]}\].

In my view the most fundamental statement of quantum mechanics is that the wavefunction, or, more generally the density matrix, represents our knowledge[1] of the system we are trying to describe.

Mermin then goes further than Peierls - while Peierls has attempted to establish strong correspondences between the density matrices of different agents, Mermin demonstrates that all that can be assumed is the very weak relationship that an agent cannot assign probability zero to a proposition to which another agent assigns probability one. That this relationship is so weak agrees entirely with the Bayesian picture that an agent should not be constrained in its beliefs by anything other than the information it has received - certainly not by another agent’s beliefs! Indeed, even this weak constraint is not required by probability theory itself. If one agent knows A while another agent knows B and both are interested in the proposition (say, the result of a physical measurement) X, it is not necessarily a contradiction if

\[
P(X|A) = 0
\]
\[
P(X|B) = 1
\]

As a proof, note that if A contains \(\neg X\) while B contains X, \(\Box\) follows immediately. Hence this constraint would appear to concern not epistemology, but

\[\text{[Peierls’ emphasis]}
\]

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ontology; it expresses that the world should not supply contradictory information to different agents existing in it. Probability theory itself is not concerned with how agents arrive at their information, only that they use it to reason consistently.

Even more explicit connections between Bayesian probability and probability in QM have been identified by [Fuchs 2002]. Fuchs considers density matrices to be nothing more than probabilities over the possible outcomes of certain measurements. They represent subjective beliefs about what will happen if we perform various operations on the relevant quantum system. [Srednicki 2003] further argues for the consideration of probability distributions over density matrices - that is, given that we are now viewing density matrices as probabilities, this suggestion amounts to considering probabilities of probabilities (as considered in [Bayes 1763] Chapter 18]). Considering wavefunctions as probabilities gives an interesting interpretation of Feynman’s path integrals [Youssouf 2001] as determining the probability that a particle will be found in a particular place by marginalising over all possible consistent paths to that point. That is, we know where the particle ends up, the particle doesn’t! This is epistemology, not teleology, against which we are warned by [Larocca 1986].

[Fuchs 2002] also shows that the somewhat mysterious notion of measurement in QM is identical to the application of Bayes’ rule. That is, when an observation is made of a quantum system, the resulting revision of density matrices reflects nothing more than the updating of our beliefs about the system. To see this, express Bayes’ rule in a slightly unfamiliar way: decompose our existing belief about hypothesis $H$ (given prior knowledge $I$) into a convex combination of posteriors for different possible measurements $d$

$$P(h|I) = \sum_d P(d|I) P(h|d, I)$$

and then pick off the posterior that corresponds to our actual observation $D$. In an identical way, measurement in QM can be expressed as the resolution of our initial density matrix $\rho$ into a convex combination of ‘posterior’ density matrices $\tilde{\rho}_d$.

$$\rho = \sum_d P(d|I) \tilde{\rho}_d$$

From which we pick the $\tilde{\rho}_D$ corresponding to the data actually observed.

It should be noted that a further adjustment for the effects of measurement on the observed system must then be incorporated. While this might seem unfamiliar from Bayesian treatment of classical systems, we would indeed have to perform such an update if our measurement process could be expected to have a physical effect on the system. Such effects are illustrated in Figure 1I - we distinguish the action of measurement from the information returned by the resulting observation. Note that all we must do to allow for such a scenario is to specify appropriate probabilities for each link in the diagram. Correctly applied probability theory is entirely capable of handling states sensitive to measurement.

As an extreme example of measurement affecting a classical system, consider the destructive testing of a material sample. Our measurement might allow us to determine what the ultimate tensile strength of the sample had been, but another update of our belief in its strength would have to be performed to reflect the fact that the sample was now lying in pieces on the laboratory floor.
Figure 1: Two Bayesian networks (see e.g. [Jensen 1996]) for measurement - $X$ and $X'$ are respectively the states before and after measurement, $\alpha$ is the choice of measurement and $A$ is the result returned by that measurement. A node is coloured if its value is known. [a] depicts observations reflecting changes wrought by measurement, while [b] depicts observations independent of such changes.

Similarly, when we ‘see’ something in a quantum system, we have to allow for the fact that even emitting a photon is a fairly traumatic event for such objects. Reassuringly, Fuchs shows that such an adjustment process disappears even for the quantum case if we consider measurements from which no physical disturbance could result. An important example is a measurement of one half of an entangled pair, in which information is gained about the other half without actually physically interacting with it. Our belief about that other half is updated without requiring any adjustment for physical influence.

The most well-known examination of possible explanations for the behaviour of such entangled pairs is provided by [Bell 1964]. Bell considers a hidden variable model as a potential resolution for the famous Einstein-Podolsky-Rosen (EPR) thought experiment [Einstein, Podolsky and Rosen 1935]. In the version Bell considers, the correlations between the observations $A$ and $B$ of an entangled pair performed by two separated experimenters are posited to be the result of some underlying hidden state variable $\lambda$. With $\alpha$ and $\beta$ defined to be the two experimenters’ respective choices of measurement, Bell’s probabilistic model takes the form depicted in Figure 2a. Bell then famously shows that no model of that form is sufficient to explain observed correlations in quantum experiments! This offers severe threats to our notions of local realism - does this mean that $A$ and $B$ can exert some sort of spooky causal influence on one another, no matter how far the experimenters are separated in space?

Figure 2: [a] Bell’s model and [b] Bell’s Model with relaxed independence assumptions.

Note that Figure 2a does explain a classical analogue [Potvin 2004] of the
EPR experiment. Consider concealing a pair of gloves within sealed boxes, before they are flown to opposite sides of Australia. The boxes are then given to the experimenters Ai-Ling in Albany and Barry in Brisbane, who measure the ‘handedness’ of the glove inside to give results $A$ and $B$ respectively. A hidden variable that now completely determines the relationships between Ai-Ling and Barry’s results is $\lambda$ : the right handed glove was sent to Albany. - given $\lambda$, learning $A$ tells us nothing more about $B$. Note that, however, this classical system does exhibit some of the ‘spooky’ behaviour sometimes ascribed to quantum weirdness. For example, without knowledge of $\lambda$, learning $A$ instantaneously changes our probability for $B$ by informing us about the common cause of both, $\lambda$ - but as [Jaynes 1989] is at pains to emphasise, this influence is purely epistemic! While the measurement of $A$ changes our state of knowledge concerning $B$, it in no way actually influences the real state of $B$ - no superluminal information transfer is required. The explanatory power of classical models for the behaviour of Bell’s system is further emphasised by [Pearl 1993, 2000].

However, Bell’s Theorem tells us that there really is more to the quantum world than that we are familiar with from classical mechanics. The intuitive model underlying Figure 25 is unable to cope with quantum systems. To [Jaynes 1988], the answer seems obvious - clearly we need to relax the quite restrictive dependency assumptions in Bell’s probabilistic model. Jaynes shows that with the more general model depicted in [26] we can more correctly reason about this quantum system, and satisfy the requirements of QM. Now, even if we know $\lambda$, $A$ and $B$ are still probabilistically dependent. [Porvin 2004] takes issue with this - surely Figure 25 is the only description of the physical influences possible under a local realist hidden variable model? To this, Jaynes would probably have agreed - indeed, his [Jaynes 1991] and our explicit goal is to attempt to formulate a theory to account for observed correlations without necessitating any spooky causal interaction between $A$ and $B$. However, the additional link relates only to logical inference, not to physical influence. Recall that such a probabilistic link does not necessarily indicate a direct physical connection, but rather the absence of some piece of knowledge, some unobserved factor excluded from the agent’s model. That is, there would appear to be some missing information in this model which, while explaining our correlations, is in some sense fundamentally unknowable to us - knowing any and all hidden variables $\lambda$ does not remove the probabilistic connection between $A$ and $B$. That is, no matter what we know, there is always something else we are somehow unable to know which influences both $A$ and $B$. This is congruent with the well-known uncertainty relationships that naturally emerge from quantum theory. The moral of EPR would seem to be simply that there are limits to the knowledge an agent can possess about a quantum system.

But in order to avoid unsatisfactory physical behaviour, we have introduced some fairly unsatisfactory probabilistic behaviour. In order to reason about quantum systems, we seem to need to consider probabilistic relationships unlike those for classical systems. [Youssuf 2001] has promoted exotic probability the-
ory for exactly this purpose. In a nutshell, this theory considers probabilities that may be negative or even complex. In order to stave off excessive eyebrow raising, note that taking probabilities as real numbers between zero and one is purely a matter of convention. \cite{Jaynes:2003} Chapter 2. This choice has proved convenient for the resulting close correspondence to experimental frequencies, but is by no means required. For example, using the semi-exotic probabilities defined as the inverse of the traditional probabilities gives a completely consistent alternative theory. As an illustration of the resulting new relationship with frequencies, if a trial has a semi-exotic probability of 3 to be successful, roughly a third of a large number of exchangeable trials would be expected to be a success. \cite{Jaynes:2003} Appendix A] further observes that the selection of real numbers to act as probabilities is built upon the desiderata of ‘universal comparability’: \( \forall A, B \), either \( P(A|I) < P(B|I) \) or \( P(A|I) > P(B|I) \) or \( P(A|I) = P(B|I) \). However, if we are unable to simultaneously know \( A \) and \( B \) due to Quantum Mechanical effects, it does not seem so desirable that we be able to directly compare their probabilities. \cite{Marlow:2000}. By lifting the requirement for universal comparability, we are free to choose an exotic probability space as the reals, the complex numbers or even the quaternions.

The concept of expressing probabilities of quantum propositions as complex numbers has a long history, which can be traced back to at least \cite{Dirac:1943}. Of course, insofar as our propositions concern real observables, traditional probability theory will continue to serve. Indeed, \cite{Yussel:2001} provides a connection between the two, and also to wavefunctions, another epistemic construct. However, the utility of exotic probability is realised if we wish to eliminate spooky ‘action-at-a-distance’ at the ontologic level. Employing traditional probability theory will force us to introduce conditional dependencies, of the kind discussed above for the EPR experiment, throughout our model if we are to express our simultaneous knowledge of quantum effects and local realism. Capturing knowledge of effects such as interference with traditional probabilities simply becomes too cumbersome. On the other hand, expressing the probability that a photon will have reflected off a mirror by incorporating an imaginary unit, for example, allows us to quite naturally model such a quantum system. The exotic probabilistic expression of a local realistic resolution of the EPR experiment can be found in \cite{Yussel:1995}.

The most pronounced way in which quantum systems behave differently to classical systems results from the presence of non-commutative observables \cite{Dirac:1943}. However, as demonstrated by \cite{Fuchs:2002}, even this does not provide a fundamental challenge to analysing such systems with probability theory. We must merely now allow for conditional relationships between observables dependent on the order in which they are observed. For example, for non-commutative observables \( A \) and \( B \) are measured at times \( s \) and \( t \), we merely have to take \( P(A_s, B_t|I) \neq P(B_t, A_s|I) \). The knowledge of the order in which measurements are made is now pertinent to our inferences, and hence should always be included in our probabilistic calculations when available. Such an order parameter forms the heart of the ontological explanation of quantum systems offered by \cite{Yussel:2001}. He proposes that a particle can be entirely explained by a hidden location state parameterised by its proper time - remarkably, location proves all that is required. We have to assert the reality of no quantities that are not directly measurable, such as spin. \cite{Fuchs:2002} is more skeptical, rightly emphasizing that most of the current body of QM seems to concern epistemic
statements. He asserts that on the basis of existing theory there is very little
that can really be concretely said about the ontology of quantum systems. It is
probably too much to hope that we will have a full understanding of the reality
of such systems any time soon. However, by use of probability theory, we can
certainly hope to characterise the clouds of uncertainty that obscure them.

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