Optimal operation of an energy management system using model predictive control and Gaussian process time-series modelling

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Abstract

This paper describes an optimal operation scheme for energy management systems (EMS) using Gaussian process (GP) forecasting and model predictive control (MPC) in the context of grid-connected microgrids with local generation, loads and storage. The main objective of the control is to minimize the cost of energy taken from the grid. The microgrid consists of a PV panel and a battery energy storage system (ESS), which are connected to a power grid and a local load via a DC bus. At each sampling time, the predictions for PV output power and load demand power are calculated, and an MPC algorithm is executed based on these predictions and a physical battery model to decide the setpoint of the battery. Simulations of two case studies, namely, a lab scale microgrid and a commercial microgrid, are presented. We compare the performance of MPC with various horizon lengths to a rule-based control strategy to demonstrate the cost reduction higher than 2 % by providing quantitative results.

Index Terms

energy storage system, energy management system, model predictive control, Gaussian process, microgrid

I. INTRODUCTION

Energy generation from renewable energy sources, including solar, wind and wave, provides electricity without significant carbon emissions, but these sources are subject to variable output depending on weather conditions. Despite their advantage of clean and low cost power generation, it is necessary to accommodate this. Typical solutions are to buffer the output using an energy storage system (ESS), or use demand side management (DSM)
through loads that can be flexibly turned up and down (such as hot water tanks), or use a controllable, but not necessarily zero carbon, generator [1].

The integration of an ESS with local power generation, for example using photovoltaic (PV) panels, and local loads, forms a microgrid, which may or may not be connected to a main utility grid. The microgrid requires an energy management system (EMS) to decide on the optimal charge and discharge set points for the ESS. The use of an EMS can result in a number of benefits including cost reduction, reduced carbon emissions and improvement of battery health compared to rule-based power management methods [2]. However, the forecasting uncertainty associated with the variable generation of renewable energy sources is a challenge for the EMS and this affects the optimal charge and discharge scheduling [3].

As a result, EMSs are recognised as a key distributed energy generation (DEG) enabling technology, and several control strategies have been suggested to enhance their performance. For example, in [4] a rolling horizon control strategy is suggested based on dynamic models of a diesel generator and power grid, together with predictions for power generation and load. A heuristic optimization based control is also studied in [5]. In [6], a rule-based optimal solution for a battery energy storage system (BESS) is presented, where the controller seeks to find the optimal current for charge and discharge using a third-order battery model for the state of charge (SoC), by penalizing deviations from the battery power reference. Unlike these results, we suggest here a novel method to operate an EMS by adopting Gaussian process (GP) time-series modelling for predicting the energy generation and the load, and model predictive control (MPC) as a strategy for the optimal operation of the ESS.

A Gaussian Process (GP) defines a distribution over a space of functions [7], [8]. A Gaussian process specifies a random function in a space, that is, a random (uncertain) variable that is a function. The random function is a Gaussian process if all of its finite dimensional distributions are Gaussian, which is specified by its mean and covariance functions. Gaussian process regression (with Gaussian noise) is tractable due to the conjugate properties of the Gaussian prior. Thus it is widely used in predicting stochastic variables, including PV power generation or solar irradiance levels [9]–[11].

The second part of our strategy, MPC, is a control technique for approximating the infinite horizon optimal control by repeatedly solving a finite horizon open-loop control problem at each sampling time-step. Feedback control can be achieved using this scheme since the current state is also updated by measuring the states of the systems at each sampling time. MPC is often used in industrial applications due to its capability to deal with constraints on input, output and the states of the systems in an explicit manner [12]. Applications of MPC in an EMS and similar areas, such as heating, ventilation and air-conditioning (HVAC) or other industrial applications have been widely studied [13], [14]. In these works, the data including weather prediction, energy consumption and mathematical models of storage systems or target buildings are supplied to the MPC controller as constraints or external signals, so that the
controllers use numerical solvers such as linear or nonlinear programming, depending on the problem formulation and modelling to solve optimization problems subject to the constraints. This type of a data-enabled MPC scheme has also been adopted within an artificial neural network to improve the efficiency of a small DEG systems and within an islanded systems equipped with weather forecasting models to plan load shedding [15], [16].

This paper describes an optimal operation scheme for an EMS using GP time-series modelling for prediction combined with MPC. The remainder of this paper consists of five parts. In Section II, the architecture of microgrids will be described with literature review. In Section III, the EMS is introduced followed by the detail of GP, the structure of our system and MPC. The mathematical model of the system is also provided here with the appropriate constraints. In the Section IV, the combination of a GP and an MPC is presented with a cost for the energy taken from power grid to minimize. A simulation result with longer predictions for output of PV arrays and local loads is also reported in Section V. Finally the conclusions and future study for further result are given in Section VI.

A. Notation

A bold face denotes a sequence of a variable from time \(0\) to \(N-1\), for example \(x = \{x(0), x(1), \ldots, x(N-1)\}\) unless a time period, such as from \(1\) to \(N\) is specified. The notation \(I_{i:j}\) is a set of integers from \(i\) to \(j\).

B. Acronyms

ESS (energy storage system), DSM (demand side management), PV (photovoltaic), BESS (battery energy storage system), HVAC (heating, ventilation and air-conditioning), MPC (model predictive control), EV (electric vehicle), PMSG (permanent magnet synchronous generators), WEC (wind energy conversion), ELDC (electrical double-layer capacitor), HIL (hardware-in-loop), GP (Gaussian process), RFB (redox flow battery), SoC (state of charge), SoE (state of energy), PCS (power conversion system), SOS (special ordered set), MPPT (maximum power point tracker)

II. MICROGRID ARCHITECTURE

The hybrid power system configuration is mainly categorized into three types; AC system, DC system and hybrid AC/DC system [17] [18]. The choice of the bus bar depends on the type of distributed generators and load to be supplied. For instance, while photovoltaic (PV) panels and batteries supply DC power, electrical generators found in diesel generators, small wind turbines and low head hydro turbines produce AC power. The research conducted for each hybrid configuration is briefly discussed. In particular, this section focuses on the lower level controls (also widely known as primary and secondary control) which were being carried out for different microgrid configurations. Besides building on the fundamental knowledge of microgrids, they highlight the key attributes of lower level control strategies which should be complemented with higher level control (EMS).
In an AC microgrid, different energy sources are connected to the AC bus via power electronic devices. If a DC storage system such as the battery bank is utilized, it is interfaced using a bidirectional power converter. The implementation of AC microgrid systems can be observed around the world [18]. From a research perspective, the study of AC microgrids was mainly in the area of droop control (load sharing), harmonic filtering, line impedance effect and stability of the overall system [19] [20] [21] [22]. The parallel operation of the inverters in an isolated scenario was found to provide improved control compared to the conventional droop control method, if the line resistance to reactance ratio is taken into consideration [19]. Similarly, another study experimentally validated that the line impedance consideration can compensate for the non-uniformity of the inverters [20]. From the control system point of view, the angle droop control has been considered as a substitute for conventional frequency droop [21]. Although it ensures proper load sharing, the overall stability is affected by the high gain angle droop control, so a supplementary control loop is required to stabilise the system for a range of operating conditions while ensuring satisfactory load sharing.

Since the concept of a smart grid was introduced, the use of DC systems for industrial power supply and commercial buildings has increased. With the rapid deployment of DC-based distributed generation (such as PV panels), the option of connecting consumer electronics devices to a DC bus system is viable. Consumer devices (computers, LED lighting and mobile phones) need DC power for their operation [23], so a conversion stage from the available AC power into DC is required, which is typically achieved using inefficient rectifiers. Moreover, the power generated from the PV systems must first be converted to AC for transmission on an existing AC bus, and then later converted back to DC for end users. These DC-AC-DC power conversion stages result in substantial energy losses and eliminating this waste could improve PV system performance by as much as 25%. Based on simulations, a year of the losses on a DC microgrid system for residential houses was evaluated and compared with the loss in an AC system, with the difference between the losses to the two systems of about 15% [10]. A distributed MPC approach has been proposed in [24] to extract optimal power from the PMSG-based WECSs in a cooperative manner, rather than competing with each other. Simulation results showed that the control scheme is especially useful in coordinating the load sharing between multiple PMSGs in a situation where the ESS is limited or runs out of capacity. A low-voltage bipolar-type DC microgrid has also been proposed to supply power via a three-wire distribution system [25]. The bipolar DC bus increases supply reliability and it allows users loads to choose the source voltage from either bus. A laboratory-scaled system has been constructed to examine the fundamental characteristics of the proposed system [25].

A hybrid scheme has both AC and DC electrical buses. In Tagajo campus of Tohoku Gakuin University, Japan, a hybrid AC/DC isolated microgrid has been developed for researchers to carry out microgrid related research and development [26]. The DC bus is integrated with PV systems, a wind turbine, a diesel generator, a variable DC
load, EVs and an energy storage system, which consists of secondary batteries and electrical double-layer capacitor (ELDC). An inverter was placed between the DC bus and AC bus to serve the AC load. Since most of the generation and loads were connected to the DC bus, the work focused on stabilising the DC voltage within an acceptable range using a ‘Coordinated Band Control’ strategy. In particular, the battery, ELDC and diesel generator play the role of stabilising the DC voltage. In another study, multiple modular bidirectional power converters were used in the hybrid AC/DC microgrid system [27]. The authors proposed a distributed coordination control method to enable the power flow between AC and DC buses in both grid-connected and islanded modes. The control strategy was verified with a real-time hardware-in-loop (HIL) simulation.

The DC microgrid structure considered in this work is demonstrated in Figure 1. The PV panels and batteries are connected to the common DC bus via bespoke DC/DC converters. The utility grid (which is connected through the point of coupling (PCC)) can be used to absorb the excess energy (grid exporting) or as a source of electricity for the microgrid, depending on the state of PV generation, load demand and the availability of battery capacity at that particular time. An inverter is required to convert the DC voltage into a usable AC voltage to supply local loads.

Fig. 1. The structure of microgrid

III. ENERGY MANAGEMENT SYSTEM

Whenever more than one energy source exists within a microgrid, an EMS is required to effectively guide the power flow within the system. Ideally, an EMS aims to maximise the use of renewable energy, reduce the stress level experienced by ESS, minimize the cost of electricity and maintain the stability and reliability of the system by supplying the load under all conditions. In general, an EMS can be implemented with conventional rule-based
strategy or intelligent-based strategy typically based on optimization algorithms. However, it is worth noting that an EMS which works well for certain microgrids may not be optimised for other configurations. In addition, it is acknowledged that there are many possibilities for an EMS to manage the power flow within a system, depending on the objectives and criteria defined by the users. This section reviews some of the EMS approaches that have been proposed for different microgrid configurations.

A microgrid with renewable energy sources is considered an effective method for DEG, especially if it is equipped with an energy storage device [4]. Installed as a component of an ESS, the basic function of an EMS is to optimize the operation of the ESS with respect to system constraints including the energy supply, demand and generation, while considering a number of factors such as economic cost, grid stability, battery health or combinations of these objectives. The term economic cost is used to describe the total cost for energy generation, conversion, storage, import/export, transmission and consumption, which covers all the expenses of energy supply and demand within the microgrid [28]. For efficient operation of the EMS with these goals, rule-based control [6] and a mathematical optimization-based control associated with models [4] are the most commonly adopted approaches. The second approach will be considered here, and in Section IV the formulation for the optimization will be introduced.

A. Structure of an on-grid microgrid with an ESS

Figure 1 presents the structure of the microgrid under consideration throughout this paper. It consists of a battery, PV arrays and a load, which are on-grid so that these devices are connected to a DC bus via DC/DC or DC/AC converters. Unlike an off-grid microgrid, which typically contains a diesel generator as a backup energy source, and the power grid is used to drain the excessive energy from PV can provide an additional source if the energy supplied by the PV does not meet the demand of the load. A power converter between the components and the DC bus transmits electrical power while adjusting the appropriate voltage and current within the sources and the load.

Figure 2 describes the structure of an EMS, which is attached to an ESS. The commands generated by the EMS are delivered to the converters via an ethernet connection and all the measured information from PV and loads are delivered to the EMS via the same path.

In addition to this hardware structure, the main algorithm of the EMS consists of two components: a GP and an MPC. The GP predicts the energy supply from the PV and the demand, based on the measured historical data. Details of the Gaussian process used for time series modelling are given in IV-A. An MPC is responsible for determining the optimal operation of the EMS with respect to the prediction generated by the GP using a mathematical optimization tool. In the Section IV-B, the mathematical formulation of the MPC will be introduced for different settings of microgrid systems.

Before we proceed to the problem formulation, in the next section III-B, we state the following simplifying assumptions used throughout this paper.
1) There is no energy loss in inverters and transmission lines.
2) Data transfer is assumed fast compared to the update rate from the controllers so the time delay between the
   EMS, energy sources and loads can be ignored.
3) This paper concentrates on the performance aspects, so the low-level controllers, such as high frequency
collectors for the converters, are not covered.

B. Problem statement

The objective of the Energy Management System (EMS) can be defined in a number of ways depending on the
needs of the customers, which usually include reduction of cost spent for backup energy sources while operating
a microgrid, and increase of battery life expectancy in an ESS while grid stability is maintained. In this paper, the
goal of our EMS is to minimize an approximation to a sum of stage cost function \( \ell(\cdot, \cdot) \) over infinite time duration

\[
\sum_{k=0}^{\infty} \ell(k, P_{\text{Grid}}(k))
\]

where \( \ell(\cdot, \cdot) \) is a cost, for example, spent for the power taken from the utility grid, whose form is \( \alpha(k)P_{\text{Grid}}(k) \) or
\( \alpha(k)|P_{\text{Grid}}(k)|^2 \) for a given unit energy price \( \alpha(k) \) at time \( k \). The infinite time duration in the summation infers that
in most cases one may not expect an EMS to stop its operation after a certain period of time. The constraints to
chief while minimizing the cost function are mostly the prediction provided by the GP and the mathematical model of the system to operate. These models include the dynamics of a battery with its operation range of the state of charge (SoC) or temperature. In the following sections, we introduce the examples of a lead acid battery model and a redox flow battery (RFB) model that is currently under development. The former is parametrized based on our laboratory-scaled microgrid [29] whilst the latter was solely used as our case study. Nevertheless, it is important to note that the proposed EMS algorithm may be applicable to other types of batteries.

C. Battery model: lead-acid battery

In this section, we introduce a mathematical model of a 12V lead acid battery, YUASA REC14-12, which is employed in a labscale microgrid system. For a charge capacity of the battery $Q$ and the battery current $I_{\text{Batt}}$, for the discrete-time model of the system presented in III-A, we define a state of charge (SoC), $0 \leq \text{SoC}(k) \leq 1$ as the ratio between the current charge over the capacity, and adopt the following Coulomb counting method:

$$\text{SoC}(k + 1) = \begin{cases} 
\text{SoC}(k) - \eta_C \frac{T_s}{Q} I_{\text{Batt}}(k): \text{Charge}, & I_{\text{Batt}}(k) \leq 0 \\
\text{SoC}(k) - \eta_D \frac{T_s}{Q} I_{\text{Batt}}(k): \text{Discharge}, & I_{\text{Batt}}(k) \geq 0,
\end{cases} \quad (1)$$

where $T_s$ is a sampling time, and $0 \leq \eta_C \leq 1$ and $0 \leq \eta_D \leq 1$ denote the charge efficiency and discharge efficiency of the battery respectively. Note that the sign of the battery current is used to indicate the direction of current flow, so the positive value implies the battery supplies energy to the DC bus while negative sign implies the reverse.

The following battery voltage equations are taken from [30], and for the sake of completeness, we give the equations here. The battery voltage varies with respect to the SoC of the battery and the exponential zone voltage $V_{\text{Exp}}(t)$, which is determined by the following dynamics:

$$\frac{dV_{\text{Exp}}(t)}{dt} = B \cdot |i(t)| \cdot ( - V_{\text{Exp}}(t) + A \cdot u(t) ) .$$

where the charge or discharge mode, $u(t)$, is 1 during charge while 0 during discharge [30]. For this $Exp(t)$, the battery voltage is different from its charge and discharge status:

$$\text{Discharge: } V_{\text{Batt}, D} = E_0 - R \cdot i - \frac{KQ}{Q - it} (it + i^*) + V_{\text{Exp}}(t) , \quad (2)$$

$$\text{Charge: } V_{\text{Batt}, C} = E_0 - R \cdot i - \frac{KQ}{it - 0.1Q} i^* - \frac{KQ}{Q - it} it + V_{\text{Exp}}(t) , \quad (3)$$

where all the variables are provided in Table I.

Since the power charge/discharge for the battery is our interest, the current-voltage relation is used to formulate the battery power $P_{\text{Batt}} = V_{\text{Batt}} I_{\text{Batt}}$. As literature including [4] suggests, in on-grid systems, the power grid is
controlled at a high frequency to compensate for the discrepancy between demand and supply from the battery and PV, or to dump the remain of PV energy that cannot be stored in the battery. This gives the power balance equation as:

\[ P_{\text{Load}}(k) = P_{\text{Batt}}(k) + P_{\text{PV}}(k) + P_{\text{Grid}}(k), \]  

which is a key relationship in the MPC formulation in Section IV.

D. Battery model: redox flow battery

For the discrete-time model of the system presented in Section III-A, we define a state of energy \( 0 \leq SoE(k) \leq 1 \) as the ratio between the currently charged energy per the capacity, adopting a following Coulomb counting approach:

\[ SoE(k + 1) = \begin{cases}  
SoE(k) - \eta_C T_s P_{\text{Batt}}(k): \text{Charge, } P_{\text{Batt}}(k) \leq 0 \\
SoE(k) - \eta_D T_s P_{\text{Batt}}(k): \text{Discharge, } P_{\text{Batt}}(k) \geq 0 
\end{cases} \]  

where \( T_s \) is a sampling time, and \( 0 \leq \eta_C \leq 1 \) and \( 0 \leq \eta_D \leq 1 \) denote the charge and discharge efficiency of the battery respectively.

An additional physical constraint on the system is that an RFB should operate in a given temperature range, and the thermal model of the battery with the temperature \( T \) is:
\[ T(k + 1) = \begin{cases} 
T(k) + \eta_C T_s P_{\text{Batt}}(k) + \eta(T_{\text{en}} - T(k)) & : \text{Charge} \\
T(k) + \eta_D T_s P_{\text{Batt}}(k) + \eta(T_{\text{en}} - T(k)) & : \text{Discharge} 
\end{cases} \] (6)

where the positive constants \( \eta_C, \eta_D \) and \( \eta \) reflect the effect of charge power, discharge power and the difference between the temperature of the battery and enclosure. Outside the temperature range, in our case between 10–40 °C, it is known that the chemical structure can be degraded so that the RFB may not operate as expected. In order to maintain consistency with the power balance equation (4), we use the conventional notation of the positive value \( P_{\text{Batt}} \) to present battery discharge while negative corresponds to charge. However, our MPC does not determine a detailed battery voltage and current in the circuit, since in the hardware used here, an external battery management system (BMS) is responsible for adjusting the voltage and current. This can be justified, since first, from the customers’ point of view, the matter of interest is power or energy supply and demand rather than the voltage or current of the battery. Secondly, the computation of voltage and current at the same time by both of PCS and EMS is unnecessary.

IV. MPC WITH GP

In this section, we describe an MPC scheme combined with the GP predictions. Unlike the usual MPC settings, prediction by a GP is important in our system since the MPC is performed with respect to the prediction. This ‘prediction’ and ‘control’ procedure is a key idea for improved performance. To implement an MPC for the system, the following procedure is repeated at every sampling time.

1) Transmit the current measurements from the PV, grid and battery to the EMS.
2) Predict the PV output and load over the next 4-hour starting from the present time.
3) Run an MPC with respect to the predictions included as constraints.
4) Transmit the set-point for the energy flow and charge/discharge of battery decided by the MPC to the inverters via the Ethernet interface.

The detail of the GP and the MPC are introduced in the following sections respectively. Should be “Two variants of MPC are proposed in Section IV-B depending on the modelling of the batteries. For the control of a lead-acid battery, we set the current as the manipulate variable and state of charge (SoC) as the states. For an RFB, on the other hand, the battery power and state of energy (SoE) are chosen as the manipulate variable and states respectively.

A. Gaussian Process for time-series modelling

We model the physical phenomena (both monitored and energy harvesting phenomena) as temporally-dependent continuous processes. We assume that these processes are temporally correlated with themselves, but are independent
from each other. Such models have recently become popular due to their mathematical tractability and accuracy [31]–[33]. The degree of the temporal correlation in the process increases as the decrease of the separation between two observing time instants decreases and can be accurately modelled as a Gaussian random field [34]–[36]. A Gaussian process (GP) defines a distribution over a space of functions and is completely specified by the equivalent of sufficient statistics for such a process (i.e. mean function $\mu(\cdot)$ and covariance function $K(\cdot, \cdot)$). It is formally defined as follows.

**Definition 1:** (Gaussian process [7], [8]): Let $\mathcal{X} \subset \mathbb{R}^D$ be some bounded domain of a d-dimensional real-valued vector space. Denote by $f(x) : \mathcal{X} \rightarrow \mathbb{R}$ a stochastic process parametrized by $x \in \mathcal{X}$. Then, the random function $f(x)$ is a Gaussian process if all its finite dimensional distributions are Gaussian, where for any $m \in \mathbb{N}$, the random variables $(f(x_1), \ldots, f(x_m))$ are normally distributed.

We can make predictions of the unknown field at unobserved time $x_*$, given a set of $N$ sensor observations $Y := [Y(x_1), \ldots, Y(x_N)]^T$, at locations $x_{1:N} = [x_1; x_2; \ldots; x_N]$.

The estimated field at this time is denoted by $\hat{f}_*$:

$$\hat{f}_* = \mu(x_*) + \text{Cov}[f(x_*), Y] \text{Cov}[Y, Y]^{-1} (Y - \mathbb{E}[Y])$$

$$= \mu(x_*) + k(x_*, x_{1:N}) (K(x_{1:N}, x_{1:N}) + \sigma_w^2 I)^{-1} (Y - \mu(x_{1:N})).$$

The uncertainty (statistical error) is given by:

$$\sigma_*^2 = \text{Cov}[f(x_*), f(x_*)] - \text{Cov}[f(x_*), Y] \text{Cov}[Y, Y]^{-1} \text{Cov}[Y, f(x_*)]$$

$$= k(x_*, x_*) - k(x_*, x_{1:N}) (K(x_{1:N}, x_{1:N}) + \sigma_w^2 I)^{-1} k(x_{1:N}, x_*).$$

In this paper, we only consider using historical PV data to predict the future PV power due to the time complexity required for the system to run (i.e., every 15 minutes). In addition, the PV data could also be considered as integration of solar irradiance within a certain time interval, since the PV information already contains the irradiance information. The improvement due to inclusion of solar irradiance prediction might not be obvious due to the high fluctuation of solar irradiance, but it will be an interesting topic to study in the future work.

Therefore, from the prediction generated by GP, we derive the predictions of power from PV and the load as follows:

$$P_{PV}(k) = \hat{P}_{PV}(k) + w_{PV}(k) \quad (7)$$

$$P_{Load}(k) = \hat{P}_{Load}(k) + w_{Load}(k). \quad (8)$$

where $\hat{\cdot}$ presents the mean of the predictions for appropriate variables and $w.$ denotes unknown prediction errors. Since the result of prediction by the GP is presented as the mean and standard deviation, we focus on the mean
value of the prediction here.

B. Model Predictive Control for a microgrid with a lead-acid battery

MPC is a real-time optimization based control technique, which accounts for all constraints in the optimization to determine the control action. The principle of MPC is to compute the behavior of the system for a finite time period, which is called the prediction horizon, and then from the solution of the prediction, the first element is applied to the system until the next sampling time. By repeating the same procedure with a finite horizon open-loop optimal control at every sampling time, we approximate the infinite horizon open-loop optimal control given in [12]. It is achieved by solving an optimization problem minimizing a desired objective function subject to constraints of state, input and output at every sampling time, based on predictions of the future power obtained for the system. The optimization problem at time $k$ based on the predictions from GP (7) and (8) is as follows:

$$
\min_{P_{\text{batt}}} \sum_{k:P_{\text{load}}(k)} \alpha(k) P_{\text{grid}}(k) \approx \min_{P_{\text{batt}}} \sum_{k:P_{\text{load}}(k)} \alpha(k) \left( \hat{P}_{\text{load}}(k) - P_{\text{batt}}(k) - \hat{P}_{\text{PV}}(k) \right). 
$$

We add several constraints to this optimization to handle the transition between charge and discharge processes. First, to incorporate the voltage characteristics and the dynamics of the battery from the charge and discharge states, as can be seen in (5) and (2) respectively, we introduce two variables to present charge and discharge current $I_{\text{Batt,C}}(k)$ and $I_{\text{Batt,D}}(k)$ respectively, such that

$$
I_{\text{Batt}}(k) = I_{\text{Batt,C}}(k) + I_{\text{Batt,D}}(k),
$$

$$
I_{\text{Batt,C}}(k) \leq 0,
$$

$$
0 \leq I_{\text{Batt,D}}(k),
$$

$$
I_{\text{Batt,C}}(k)I_{\text{Batt,D}}(k) = 0,
$$

The constraint in (14) ensures that only one of the two between $I_{\text{Batt,C}}(k)$ and $I_{\text{Batt,D}}(k)$ can be non-zero at the same time. Using this formulation, the dynamics in (5) become

$$
SoC(k + 1) = SoC(k) - \eta_C \frac{T_s}{Q} I_{\text{Batt,C}}(k) - \eta_D \frac{T_s}{Q} I_{\text{Batt,D}}(k),
$$

We now present the optimization problem as follows:
Problem 1: MPC for a microgrid with a lead-acid battery

\[
\min_{I_{Batt,C}, I_{Batt,D}} \sum_{k=0}^{N-1} \alpha(k) \| \hat{P}_{\text{Load}}(k) - P_{\text{Batt}}(k) - \hat{P}_{\text{PV}}(k) \|^2
\]
subject to

(16a)

\[
\text{SoC}(k+1) = \text{SoC}(k) - \frac{T_s}{Q} I_{Batt,C}(k) - \frac{T_s}{Q} I_{Batt,D}(k),
\]

(16b)

\[
V_{\text{Exp}}(k+1) = V_{\text{Exp}}(k) + T_s \dot{V}_{\text{Exp}}(k),
\]

(16c)

\[
P_{\text{Batt}}(k) = I_{Batt,D}(k) V_{Batt,D}(k) + I_{Batt,C}(k) V_{Batt,C}(k),
\]

(16d)

\[
I_{Batt,C}(k) I_{Batt,D}(k) = 0,
\]

(16e)

\[-2 \leq I_{Batt,C}(k) \leq 0, \quad 0 \leq I_{Batt,D}(k) \leq 2, \quad 0.2 \leq \text{SoC}(k) \leq 0.8, \quad -20 \leq P_{\text{Batt}}(k) \leq 20.
\]

(16f)  
(16g)  
(16h)  
(16i)

and we denote its solution by \( I_{\text{Batt}}^*(\hat{P}_{\text{Load}}, \hat{P}_{\text{PV}}) := \{I_{\text{Batt}}(0), I_{\text{Batt}}(1), I_{\text{Batt}}(2), \ldots, I_{\text{Batt}}(N-1)\} \). The transition between charge and discharge is incorporated in (16b) and (16e). Note that due to the complexity of the battery model, we could only minimize the approximation with 2-norm, which approximates the original objective function. The numerical solver \texttt{fmincon} in the Optimization Toolbox of MATLAB\textsuperscript{©} is used for this computation. Once the solution to the constrained minimization has been found, we apply only the first element \( \kappa(\hat{P}_{\text{Load}}, \hat{P}_{\text{PV}}) := P_{\text{Batt}}(0) \) to the system with push-and-hold fashion until the next sampling time.

C. MPC for a microgrid with an RFB (redox flow battery)

We also formulate a similar formulation applies for the microgrid with RFB. Since the RFB model, which is provided as a constraint in the optimization, adopted here is linear, the original objective function can be minimized using a mixed integer bilinear solver. To solve the mixed integer program with with Special ordered set 1 (SOS1) constraint, IBM Cplex\textsuperscript{©} is used.

The optimization is formulated to extract the cost for the power taken from the grid, but exclude selling. Since the focus of this work is on the performance of the proposed GP-MPC algorithm, the latter is omitted as it is may influence the decision variable (the charging and discharging of batteries). We divide the grid power into two as follows.

\[
P_{\text{Grid}}(k) = P_{\text{Grid}}^+(k) + P_{\text{Grid}}^-(k),
\]

(17)

in which \( P_{\text{Grid}}^+(k) \geq 0 \) is the power taken from the grid and \( P_{\text{Grid}}^-(k) \leq 0 \) is the power sent to the grid. By using SOS1 constraints, either one of the two is allowed to be assigned a nonzero value at the same time. We introduce
the following mixed-integer programming to minimize the accumulated sum of $P_{\text{Grid}}^-(k)$ multiplied by the unit price of the electricity as shown in (18a).

**Problem 2:** MPC for a microgrid with a redox flow battery

\[
\min_{P_{\text{Batt,C}}, P_{\text{Batt,D}}, P_{\text{Grid}}^+, P_{\text{Grid}}^-} \sum_{k=0}^{N-1} \alpha(k) P_{\text{Grid}}^+(k) \quad \text{subject to} \quad (18a)
\]

\[
\text{SoE}(k+1) = \text{SoE}(k) - \eta_C \frac{T_s}{Q} P_{\text{Batt,C}} - \eta_D \frac{T_s}{Q} P_{\text{Batt,D}},
\]

\[
T(k+1) = T(k) + \eta^T(T_{\text{en}}(k) - T(k)) + \eta_C^T T_s P_{\text{Batt,C}}(k) + \eta_D^T T_s P_{\text{Batt,D}}(k),
\]

\[
P_{\text{Batt}}(k) = P_{\text{Batt,C}}(k) + P_{\text{Batt,D}}(k),
\]

\[
P_{\text{Load}}(k) = P_{\text{PV}}(k) + P_{\text{Batt,C}}(k) + P_{\text{Batt,D}}(k) + P_{\text{Grid}}^+(k) + P_{\text{Grid}}^-(k),
\]

\[
0.3 \leq \text{SoE}(k) \leq 0.8,
\]

\[-5 \leq P_{\text{Batt,C}}(k) \leq 0, \quad 0 \leq P_{\text{Batt,D}}(k) \leq 5,
\]

\[
P_{\text{Batt,C}}(k), P_{\text{Batt,D}}(k) \text{ are in an SOS1 constraint.}
\]

\[
P_{\text{Grid}}^-(k) \leq 0, \quad 0 \leq P_{\text{Grid}}^+(k),
\]

\[
P_{\text{Grid}}^+(k), P_{\text{Grid}}^-(k) \text{ are in an SOS1 constraint.}
\]

\[
10 \leq T(k) \leq 40.
\]

**Remark 1:** The bilinear constraint (16e) in the optimization problem for MPC may cause difficulties in finding the global optimum especially in a short computation time allowed. In this case, we accept a suboptimal MPC with local optimum at each time step. Also, in order to reduce the computation time and achieve a better solution, a warm-start is adopted from the solution of the previous time step. The warm-start we adopt is constructed from the solution by shifting a time-step and concatenating by zero.

**Remark 2:** By replacing the stage cost function of (18a) by $\alpha^+(k) P_{\text{Grid}}^+(k) + \alpha^-(k) P_{\text{Grid}}^-(k)$, where $\alpha^+(k)$ and $\alpha^-(k)$ are unit price for buying and selling power respectively, we can implement a more general case. Throughout this paper we assume $\alpha^+(k) = \alpha(k)$ and $\alpha^-(k) = 0$, which implies that only cost for buying energy from the grid is of interest.

**Remark 3:** The key idea of this MPC formulation addressed here is the constraints (16e) and (18h) which describe the transition between charge and discharge dynamics of the battery.

**D. Decision of the horizon length $N$**

When solving the optimization problem for MPC, it is noticeable that the parameters in the optimization problem for MPC (16) that significantly affect the performance of the optimization are measured or given values from the
mathematical models of battery and physical constraints of the system. This is a well-known issue in the MPC community [37] since an MPC is a strategy which approximates the infinite-horizon continuous time domain optimal control by repeatedly solving a finite-horizon discrete-time open-loop optimal control [12]. In a standard MPC setting, we may conclude that an MPC with the longer horizon and shorter sampling time is better as long as the optimization is completed within a short period of time enough to be considered as a “real-time” [12] with the limit of computation power. In our setting, however, increasing the prediction horizon reduces the accuracy of the prediction, so there is a trade-off between control horizon and prediction accuracy. Suppose that we launch our system at $t_0$ with $N$. This implies that the two GPs are called for PV and load predictions, and provide the generated predictions to the MPC module. With a slight abuse of the notation, we denote $\hat{P}_{PV}(t|t_0)$ as the predicted PV power at $t$ predicted at $t_0$. Comparing $\hat{P}_{PV}(t_1|t_0)$ and $\hat{P}_{PV}(t_2|t_0)$ for $t_1 > t_2$, one can expect the accuracy at $t_2$ is more accurate since $t_2$ is closer to the current moment $t_0$.

In this context, there is a trade-off between a longer horizon, which provides solutions that are closer to the solution of the infinite horizon problem, and a shorter horizon, which has more accurate prediction. Also, since the number of the optimization variables in (16) increases exponentially as the horizon does, the computational load also increases with increasing horizon. We examine simulations with respect to different horizons in Section V.

V. CASE STUDY

In this section, we present two simulation results that demonstrate the performance of the control scheme presented in this paper. An operation of a labscale microgrid with a lead acid battery is presented in at the first part of this section, followed by a large-scale commercial version of a microgrid equipped with an RFB.

A. Dataset for PV and load power

The PV and load data are obtained from a Belgian electricity supplier Elia (http://www.elia.be) and scaled accordingly. The data for 7 days from the midday of 1st July 2016 to 8th July 2016 is used for this simulation with the data of 7 days prior to the starting day, which is used by the GP to predict the PV output and load power. This dataset is presented in Figure 3 with the PV and load power for the next 7 days from the starting date. The PV output power, which is presented in blue, ranges between zero and 50 kW while the load power, marked by a red line, lies between 20 and 30. It can be seen that around midday the PV power exceeds the load power.

With the given dataset in Figure 3, the prediction is performed using the Gaussian Process time-series algorithm, where the same algorithm is adopted to predict both the PV output and the load power. We display the prediction result at the first sampling time, 12:00, 1st June, in Figure 4. Compared to the load power, the GP tends to have higher error at the peak value, which is caused by a rapid increase in the data near the peak. For example in the midday of 4th July and 6th July, the sudden increase or decrease is not recognized by the prediction algorithm.
Fig. 3. Dataset for learning and the actual data for the prediction (in the shaded area)

until the next sampling time. Compared to the PV prediction, the load prediction is less affected by this change, since at any time the power is distributed between 20 and 30 kW which is a narrower range compared to the PV output. The advantage of GP prediction is that it learns the pattern from the historical data in a nonparametric way accurately. Another advantage of GP prediction is that it can also provide an estimate of the prediction uncertainty. The disadvantage of GP is that it is difficult to predict sudden increase or drop of the PV or load power, although this is also a difficult problem to other estimators.

Fig. 4. Prediction result of PV output and load
B. A laboratory-scaled microgrid with a lead acid battery

The simulation results for the following three cases are given to compare the performance of control schemes: MPC using the perfect prediction; MPC using the prediction; and for comparison purposes a rule-based operation. For the first two cases, various horizon lengths are tested in order to determine the optimal horizon length. For the rule-based algorithm, the battery charges when the output of PV is greater than the load demand, otherwise it discharges while containing the SoC and temperature within the operation range.

For all MPC simulations, the same parameters $T_s = 0.25$ (hour), the prediction horizon $N = 4, 8, 16$, (corresponding to 1, 2, 4 hours, respectively), and data of four days for learning and prediction are used. The cost function is based on the Economy 7 tariff, used in the UK, where the off-peak price is more or less half the usual electricity price and this off-peak price of the electricity is contained in the definition of objective function, so that the unit price $\alpha(\cdot)$ of electricity is 14p/kWh from 7am to 11pm and 6p/kWh from 10pm to 6am*, so that the cost for the energy taken from the power grid is:

$$\sum_{P_{\text{Grid}}(k) \geq 0, k \in T_{\text{fin}}} \alpha(k) P_{\text{Grid}}(k),$$  \hspace{1cm} (19)

where $\alpha(k) = 6$ from 10pm to 6am, otherwise 14.

The result of MPC for seven days is illustrated in Figure 5. As the prediction horizon length is varied from 4 to 16, (corresponding to 1 and 4 hours respectively) we notice a significant change in the control action when the horizon is increased from 8 to 16. (Note that a positive sign of the battery power represents discharge while a negative signed denotes charge.) The battery is mostly charged while the PV output power is higher than the load, but at the last part of every off-peak hour, it is also charged since it is beneficial to pay the off-peak price rather than peak price as long as the ESS has available capacity to store energy from any energy source.

<table>
<thead>
<tr>
<th>Horizon Length (hour)</th>
<th>4 (Ideal)</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>(Rule-based)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (£)</td>
<td>6.93</td>
<td>9.23</td>
<td>8.11</td>
<td>7.39</td>
<td>7.58</td>
</tr>
</tbody>
</table>

Table III

Cost comparison between 4 control schemes (lead-acid battery)

Table V-B represents the cost for the power taken from the grid in the following, which is a main result of this paper. The table shows that if we increase the horizon length $N$, the performance improves also. With up to 4-hour prediction and control horizon, we can benefit 2.5% of the cost spent for the energy from the grid. The simulation also shows that we can benefit 8.6% if the prediction is perfect, which is illustrated in Figure 6. The figure describes the charge and discharge power of the battery between 2nd and 3rd of July. At the end of 2nd before the solar panel generates energy, the battery starts to charge from the utility grid. As the control horizon is increased the charging

*The tariff may differ from electricity suppliers.
Fig. 5. Battery power of lead-acid battery controlled by MPC. Shaded areas denote off-peak hours.

Fig. 6. Charge/discharge profile comparison between various lengths of control horizon (Lead-acid battery)

duration is increased. The MPC with longer horizon can detect the change of the electricity earlier, and the cost for the longer term is taken into account for the optimization while the MPC with shorter horizon only consider the electricity cost in the shorter period ahead.
C. A commercial microgrid with a redox flow battery

A second case study of a microgrid with a redox flow battery is illustrated in this section, based on the microgrid in Fig. 1. This system consists of a PV panel, battery with an ESS and AC load, which are connected to a common DC bus and a power grid. DC/AC and DC/DC converters are installed to convert electrical power from one unit to the other so that frequency, voltage and current are matched throughout the load and supply of the system. We use the same PV power generation and load power profiles used in Section V-B, but differently scaled. A similar test in Section V-B is also carried out with respect to the objective function (10) for as the total energy taken from the grid.

The unit electricity price $\alpha(\cdot)$ in the objective function is determined by the following table IV of KEPCO (Korea Electric Power Corporation†).

<table>
<thead>
<tr>
<th>Classification</th>
<th>Time</th>
<th>Price (KRW/kWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>off-peak</td>
<td>23:00 – 09:00</td>
<td>59.5</td>
</tr>
<tr>
<td>mid-peak</td>
<td>09:00 – 10:00</td>
<td>112.4</td>
</tr>
<tr>
<td></td>
<td>12:00 – 13:00</td>
<td></td>
</tr>
<tr>
<td></td>
<td>17:00 – 23:00</td>
<td></td>
</tr>
<tr>
<td>on-peak</td>
<td>10:00 – 12:00</td>
<td>193.3</td>
</tr>
<tr>
<td></td>
<td>13:00 – 17:00</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE IV**

Unit electricity price for the RFB microgrid

The same MPC strategy as in V-B is adopted, with an additional constraint for the battery temperature is limited during the operation. Therefore, these charge and discharge processes are performed while the SoE and the temperature inside the battery lies within the desired operation range, which are $0.3 \leq \text{SoE} \leq 0.8$ and $10 \leq T \leq 40$. In some RFBs the enclosure temperature is cooled down by a chiller with an external power source, but our object is maintain the temperature within the range without a chiller which requires additional energy consumption.

For the MPC setup (18), the same sampling time $T_s = 0.25$ (hour) and data of seven days for learning and prediction are maintained, while the prediction horizon is varied as $N = 4, 8, 16, 32, 48, 96$. The ambient temperature is taken to be $25^\circ C$, and the result shows that the battery temperature is not a major constraint for the operation.

The following Table V-C addresses the comparison of the operation strategies. As expected, the ideal case with the perfect prediction for 24-hour achieves the best performance compared to the others, while the rule-based control technique results in the highest cost. Using MPC with the actual prediction, the cost reduction for the energy taken from the grid is about 0.88% of the rule-based algorithm, but a saving up to 1.33%. The result also shows the

†General Service(B), High-Voltage C, option 1 in http://cyber.kepco.co.kr/ckepco/front/jsp/CY/E/E/CYEEHP00202.jsp is choosen.
performance is achieved best in the case of 4-hour prediction rather than 24-hour prediction, which is clue to the increased uncertainty in the prediction as discussed earlier in Section IV-D.

<table>
<thead>
<tr>
<th>Horizon (hr)</th>
<th>24 (Ideal)</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>24</th>
<th>(Rule-based)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost (KRW‡)</td>
<td>191,355</td>
<td>196,494</td>
<td>193,337</td>
<td>192,242</td>
<td>193,284</td>
<td>193,457</td>
<td>193,866</td>
<td>193,951</td>
</tr>
</tbody>
</table>

**TABLE V**

COST COMPARISON BETWEEN 4 CONTROL SCHEMES (RFB)

Figure 3 illustrates that the data for the PV output and load, followed by the measured data on the 8th day on the shaded area. The load, which is marked in blue, has low uncertainty, while the PV output, which is marked in red, has high uncertainty. The prediction of both cases therefore shows a significant different performance. The comparison between the measured data on 8th day and its prediction result is presented in Figure 4. The difference between the two predictions is noticeable (red). When we compare the PV output prediction (blue) shade with respect to the actual PV data in red, the load prediction in a blue line is much more accurate.

With respect to the prediction given by the GP, the MPC determines how high charge or discharge of the battery is performed, as is shown in Figure 7 and 8. It is noticeable that the battery is charged during the off-peak hours with the grid power as well as the day-time with PV. It also describes the thermal characteristics of the battery, whose temperature rises during the discharge while falls while charge is in operation.

A detail of the behaviour comparison between various control horizons are illustrated in Figure 8. The darkly shaded, lightly shaded and white areas represent off-peak, mid-peak and on-peak hours respectively. Because of the difference of the time classification (off-peak, mid-peak and on-peak hours) of KEPCO, more complicated behavior is observed throughout the daytime. Similar to the behavior of MPC for the UK tariff, at the end of the off-peak hours the battery tends to charge by using the cheap electricity price. However, the existence of mid-peak hours prior to the second off-peak hour shows a significant difference. The battery is fully discharged before the time reaches to the off-peak hours, when the price is lowest. The two mid-peak hours during the daytime do not have a significant effect, since the power generated by PV is much higher than the load power.

**Remark 4:** The performance of the microgrid systems may significantly depend on the environment or the scale of the hardware as well as the computation of the optimization. For instance, if the energy generated by PV is always lower than the load, the battery is charged only during the off-peak hours by the utility grid since energy loss exists during charge and discharge processes. On the contrary, if the total sum of the energy generated by PV exceeds the amount of total load and the capacity of the battery is large enough to store all the energy during the daytime, the rule-based algorithm can operate the system optimally without the energy taken from the utility grid. Therefore, we would like to stress that the decision of the size of the battery and the PV panels is essential for the performance improvement by the optimization-based control scheme.
In this paper, we present a scheme for the optimal operation of an energy management system (EMS) controlled by a model predictive control (MPC) incorporating predictions using a Gaussian process (GP) time series model. The main idea of this strategy is to predict the load and PV power generation based on the past data, followed by minimization of the cost with respect to those predictions. Since the GP prediction is combined with MPC, which is implemented with in a receding horizon manner, the cost spent for the energy taken from the power grid is minimized in a real-time. Simulation results suggest that even with highly uncertain PV energy generation, the proposed scheme overperforms a rule-based heuristic technique. However, the benefit resulting from this optimization-based control may vary with respect to the conditions such as operation environment or the electricity tariff. The battery capacity as well as the the maximum power generation that obtained from the PV panels is also a key factor in determining the level of benefit from the algorithm presented in this paper. An extension of this paper will present the experimental results for a network of coupled ESSs, and we expect that improved sharing information between ESS’s will increase the accuracy of prediction, resulting in improved performance.

VII. ACKNOWLEDGEMENTS

This work was supported by the Korea Institute of Energy Technology Evaluation and Planning (KETEP) and the Ministry of Trade, Industry & Energy (MOTIE) of the Republic of Korea (No. 20148510011150).
Fig. 8. Charge/discharge profile comparison between various lengths of control horizon (RFB)

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