Sparse CRF: Defined on a set of random variables \( X = \{ x_1, \ldots, x_N \} \), where each random variable \( x_i \) takes a label \( y_i \in \mathbb{L} \).

- Captures long-range interactions and provides fine grained segmentations [3].
- Difficulty: There are \( |\mathbb{O}| \) number of variables \( (n = 10^6) \) so even the energy computation is Intractable.

Gaussian pairwise potentials:

\[
E(x) = \sum_{i=1}^{N} \log(1 + e^{-|x_i - y_i|})
\]

Where \( |\mathbb{L}| \leq n \).

- Approximate the above computation using the filtering method [1] \( \approx \) \( O(|\mathbb{O}|) \) computations.

Existing efficient algorithms:
- Mean Field (MF) [5]. Quadratic Programming (QP) [1] and Difference of Convex (DC) Programming [3].
- All these algorithms rely on the efficient filtering method and have linear time complexity per iteration.

Drawbacks: No multiplicative bound \( \mathcal{O} \) the solution obtained by these algorithms can be far be from the optimum.

- Linear Programming (LP) relaxation provides the best multiplicative bound but the existing algorithm is too slow.

Linear Programming (LP) Relaxation

- Introduce indicator variables: \( y_{ij} = 1 \Leftrightarrow x_i = y_j \).
- For integer labelings \( E(y) = \text{Ex}(y) \).

\[
\min_{y \in \mathbb{L}^N} \mathbb{E}(y) = \sum_{i=1}^{N} \log(1 + e^{-|x_i - y_j|})
\]

S.t.: \( y \in \mathbb{L} \), and \( \sum_{i=1}^{N} y_i = N \).

\( \mathcal{O} \) provides an integrality gap of 2 [4], and no better relaxation can be designed due to the UGC hardness result [7].

Optimizing over \( \beta \) and \( \gamma \):

- \( \beta \): Unconstrained \( \Rightarrow \) \( \beta \) is determined by the current iterate.
- \( \gamma \): Unconstrained \( \Rightarrow \) \( \gamma \) is determined by the current iterate.

Optimizing \( \alpha \):

- The conditional gradient has the same form as the subgradient of LP [2].
- The optimal step size can be computed analytically.

\[
\alpha \text{ always appears in the product } \mathbf{A} \Rightarrow \text{Space complexity is linear.}
\]

Discussion:

- We have introduced an efficient and accurate algorithm for energy minimization in dense CRFs.
- Our algorithm can be incorporated in an end-to-end learning framework to further improve the accuracy of deep semantic segmentation architectures.

Code: https://github.com/our-group/DenseCRF