\textbf{A3(b).} Various alternative forms include

\[ \frac{1}{2} \cos(2t + \phi) \quad \text{where} \quad \phi = \tan^{-1} 2 - \pi = -2.034 \]

and

\[-\frac{1}{2\sqrt{5}} \cos 2t + \frac{1}{\sqrt{5}} \sin 2t \]

\textbf{A5.} With \( S = \begin{bmatrix} 5/8 & 3/8 \\ 3/8 & 5/8 \end{bmatrix} \) you get an ellipse rotated the other way to the one sketched in the solution. While this still counts as a valid answer to the question, the matrix corresponding to the ellipse as sketched, i.e. with its major axis parallel to \( y = x \), is in fact \( S = \begin{bmatrix} 5/8 & -3/8 \\ -3/8 & 5/8 \end{bmatrix} \).

Check: from the sketch it is clear that \( x = \begin{bmatrix} \sqrt{2} \\ \sqrt{2} \end{bmatrix} \) should satisfy \( x^T S x = 1 \), which it does with the latter \( S \), but not the former. The bug in the working occurs in the second last line: although \( R^T \) has been evaluated correctly, there is a mixup between \( R \) and \( R^T \) when substituting into the expression for \( S \) at the top of the page.

\textbf{A7.} The correct solution is

\[ \text{Re} = \sin(r \cos \theta) \cosh(r \sin \theta) , \quad \text{Im} = \cos(r \cos \theta) \sinh(r \sin \theta) \]

No negatives anywhere! To cut the manipulations is better just to go

\[ \sin(re^{i\theta}) = \sin(r \cos \theta + rj \sin \theta) \]
\[ = \sin(r \cos \theta) \cos(rj \sin \theta) + \cos(r \cos \theta) \sin(rj \sin \theta) \]
\[ = \sin(r \cos \theta) \cosh(r \sin \theta) + j \cos(r \cos \theta) \sinh(r \sin \theta) \]

using formulae \( \sin jx = j \sinh x \) and \( \cos jx = \cosh x \) from HLT p. 7.

\textbf{B1(a).} In the solution there are various issues with signs, e.g. \( \frac{d}{dx} (\cos^{-1} x) = \frac{-1}{\sqrt{1 - x^2}} \) not \( \frac{1}{\sqrt{1 - x^2}} \), but happily the errors seem to cancel.

\textbf{B2(a).} Another possibility for \( J = \cosh^2 u - \cos^2 v \).
B2(d). Working out the double integral gives

\[ 1 + \frac{\pi}{2} \cosh 1 \sinh 1 = 3.849 \]

which looks believable since a sketch of the region (in x-y space) shows the area must be around \( 2 \times 2 = 4 \). The deleted typed solution, \((\sinh 2 - 2)(\pi + 2)/8\), works out to 1.046, which is clearly some way out.

B3. The equation in the last line of the solution is correct; taking the final step and solving it for \( y \) gives the two possible solutions to the DE as \( y = 5 - x \pm \sqrt{2}x^2 - 12x + \text{const} \).

B5(c). There is a minor typo (not carried through) in the working for \( b_k \): \( \frac{4}{T} \) should be \( \frac{8}{T} \).