A1. The derivatives are given correctly, but the suggested formulae don’t reproduce them. Instead we want something like

\[ 3^n \left( (-1)^{\frac{n-1}{2}} \sin 3x \quad \text{n odd} \right) \]
\[ 3^n \left( (-1)^{\frac{n+2}{2}} \cos 3x \quad \text{n even} \right) \]

or preferably a formula that works for all \( n \), such as

\[ -3^n \cos \left( 3x + \frac{n\pi}{2} \right) \]

A3. The arg is out by 180\(^\circ\) (it should be \( \phi = +129.8^\circ \) rather than \( \phi = -50.2^\circ \)).

A6. The question should presumably say “zero for \( t = 5 \text{ s} \)”. The sketch in the solution could also be better, though the students weren’t asked for this.

B1(b). The correct answer is \( \bar{z} = 25a/48 = 0.5208a \) (= 0.2708a above the truncation level).

The error is in the calculation of \( V \) at the bottom of p. 6: \( (3a/4)^3 = 27a^3/64 \) not \( a^3/64 \), so \( V = 1.325a^3 \) not 1.751a\(^3\). Note that the formula for the centre of mass of a sphere segment on HLT p. 28 is also wrong, though interestingly it does give the correct result for a hemisphere (and a remarkably good approximation for truncations that retain less than a hemisphere).

B2(c). There is a mistake in the fourth line of working: the result of the integral w.r.t. \( v \) is \( u^2v + v^3/3 \) (not \( u^2v - v^3/3 \)). This gives the volume as 70/3 = 23.33 (not 5). Verify with a direct calculation in the \( x-y \) plane:

\[ A = \int_{y=1}^{2} \int_{x=0}^{\sqrt{\frac{1}{2}}} dy dx + \int_{y=2}^{4} \int_{x=0}^{\sqrt{\frac{y^2}{8}}} dy dx = \frac{2}{3} + \frac{5}{3} = \frac{7}{3} \quad \therefore \quad V = \frac{70}{3} \]

Check: the point labelled c in the solution has coords (1.5, 2) so the shading approximates a triangle of area \( 0.5 \times 3 \times 1.5 = 2.25 \). We therefore expect the volume to come out around 22.5, which it does.
B3. There seems to be confusion over whether \( a = 3j \), as stated in the question, or \( a = 3 \). The answer given to part (a) is correct for \( a = 3j \). The eigenvalues obtained in part (b) are the negative of the eigenvalues for \( a = 3 \), which are

\[-1.216 - 3.684j, \ 9.216 + 1.684j\]

The bug in the working is in the solution of the quadratic, but this is not really relevant. It is necessary to rework part (b) completely with \( a = 3j \), giving the eigenvalues

\[7.326 + 2.313j, \ -2.326 - 1.313j\]

B4(b). The partial fractions can be done by inspection using the cover-up rule (much quicker than comparing coefficients). Also, the final statement of the answer is a little loose, since it’s not just the constants that kick in at \( t = 5, 15 \) and \( 20 \) s. It would be preferable to say

\[V(t) = g(t) + 2g(t - 5) - 4g(t - 15) + g(t - 20)\]

where \( g(t) = \frac{1}{8} \left( 1 - 2e^{-2t} + e^{-3t} \right) \cdot u(t) \)

B5(c). If taking \( g = 9.81 \text{ m/s}^2 \) the answer comes out a little lower (5.61 m/s).

B5(d). The question asks for the velocity at \( t = 1 \) s, not \( t = 1.5 \) s, so the required answer is 6.03 m/s (or 5.96 m/s if taking \( g = 9.81 \text{ m/s}^2 \)).

B7(c). Minor typo in penultimate line of working (should be \( x^2/2 \)). Also, the answer is 74.6 not 75.6.