Relaxed Matching Kernels
Andrea Vedaldi and Stefano Soatto - UCLA Vision Lab

Kernels for Recognition

Goal: capture systematically recent kernels for bag-of-features representations which exploit spatial information. Introduce new kernels.

Beyond Bag of Features

- Visual features
  - Locations \(l_1, \ldots, l_N \in \mathbb{R}^2\)
  - Descriptors \(d_1, \ldots, d_N \in \mathcal{F}\)
  - Quantization (visual words) \(b_1, \ldots, b_N \in B\)
- Bag-of-features
  histograms of visual words: \(h^k(b), k = 1, 2\)

On the Base Kernel

How do we choose the base kernel \(K(h^1, h^2)\)?

\([19]\) introduced a large family of kernels for probability distributions that can readily be used in the RMK framework.

\[
K(h^1, h^2) = \sum_{b \in B} \min\{h^1(b), h^2(b)\}
\]

Comparision

- L1 kernel \(k(a, b) = \min\{a, b\}\)
- Chi2 kernel \(k(a, b) = (2(ab)/(a+b)\)
- Hellinger's kernel \(k(a, b) = \sqrt{ab}\)

Radial Basis Function versions of all RMKs are defined up to a scaling parameter

\[
K_{\text{RBF}}(I^1, I^2) = \exp(-\alpha K(I^1, I^2))
\]

Lemma: All such base kernels yield positive definite (PD) RMKs. The RBF versions are PD as well.

On the Weights

- Are relaxations redundant?
- Are we double-counting features?
- What is the meaning of the weights?

Theorem. \(F_r\) is a non decreasing function of the relaxation order \(r\) for all choices of the base kernel.

\(F_r\) can be thought as a distribution over relaxations and an RMK as the expected values of the weights \(w\). RMKs can also be rewritten as:

\[
K(I^1, I^2) = \sum_{r=0}^{R-1} w_r F_r = \sum_{r=0}^{R-1} (W_{R-1} - W_{r-1}) f_r
\]

- \(f_r = F_r - F_{r-1}\) is the variation of the similarity score at level \(r\)

\[
W_r = \sum_{q=0}^{r} w_q
\]

are the integral weights

\[
W_{R-1} - W_{r-1}\] decreases monotonically to zero

Interpretation: An RMK searches for the smallest relaxation order for which the data match well.

Efficient Calculation

All RMKs can be efficiently computed by a single pass through finest quantization level.

Key idea: Visit bins by traversing all visual words once. This is possible because visual words are organized hierarchically.

New RMKs

Graph Matching Kernels (GMK). Features are often arranged in graphical configurations. GMKs compare graphs of visual words which match coarsely.

- Features: pairs of visual words at graph distance less than \(\tau\)
- Matching: count how many similar pairs there are.

\[
F_r = \sum_{(d_i, d_j) \in B_r} k(h^1_i(d_i, d_j, \rho), h^2_j(d_i, d_j, \rho))
\]

Observation: If the nodes have unique names (visual words), then a GMK is zero if and only if, the graphs are identical.

Agglomerative Information Bottleneck Kernels (AIBMK). Similar to PMK, but it creates hierarchy based on AIB.

Experiments

GMK for matching graphlets of features.

Conclusions

- RMKs generalize previous matching kernels for image comparison.
- RMKs highlight common properties and provide an universal algorithm.
- Careful experimentation reveals that current formulations may be insufficient to exploit spatial information.