The information available to a moving observer from specularity

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This paper examines the information available from the motion of specularity (highlights) due to known movements by the viewer. In particular two new results are presented. First, it is shown that for local viewer movements the concave/convex surface ambiguity can be resolved without knowledge of the light source position. Second, the authors investigate what further geometrical information is obtained under extended viewer movements, from tracked motion of a specularity. The reflecting surface is shown to be constrained to coincide with a certain curve. However, there is some ambiguity – the curve is a member of a one-parameter family. Fixing one point uniquely determines the curve.

Keywords: specular motion, convex/concave test, viewer motion

One of the aims of computer vision is to extract concise surface descriptions from several images of a scene. The descriptions can be used for the purposes of object recognition and also for geometric reasoning (such as collision avoidance). Stereo vision determines depths at surface features (such as edges and creases), often only sparsely distributed. It cannot yield full information on surface shape.

Ambiguity arises when the distribution of surface elements is very sparse. This is common with smooth, especially manmade, objects. Typically the only visible surface features will be contours (steps or creases) between adjacent surfaces. Apart from at contours, surface shading varies smoothly making stereo correspondence difficult. Judicious analysis of surface shading can considerably augment the geometric information obtained directly from stereo vision.

Ikeuchi\textsuperscript{1} has described methods of finding the surface normal (and hence the depth) at every point of a smooth surface by using shading and a bounding stereoscopically viewed contour. However, 'shape from shading' algorithms of this type depend on having precise photometric information about the light source and surface reflectance properties. This is not possible except in a strictly controlled environment.

An alternative is to use more qualitative methods which do not return the depth at every point\textsuperscript{2}. There are a number of shading cues which can provide robust and reliable information about surface shape and source position. For example, specularity (highlights), shadow boundaries and self-shadowing can yield considerable information on local surface curvature and source position. Similarly extremal boundaries constrain the local curvature of the surface (specifically the Gaussian curvature) and extrema of intensity can be related to surface characteristics\textsuperscript{3}.

This paper addresses the question 'What information is available from observing the movement of specular points in two or more images for known viewer motion?' The detection of specularity\textsuperscript{4–6} is not described. No surface characteristics are assumed\textsuperscript{5–8} other than the simple mirror condition. Where necessary a point light source is assumed. This is not a restriction because if the source has finite extent then the brightest point of the specularity can be used. The information contained in the shape or intensity profile of the specularity\textsuperscript{8–10} is not used. A brief review of two approaches for deriving surface shape from the movement of specularity follows in the next section.

Two new results are presented: in the third section a simple test which resolves the convex/concave ambiguity is described. All that is required is two views of the scene and an estimate of viewer-surface distance. No knowledge of the light source position or surface slant is needed. The implementation of this test is demonstrated on synthetic and real images in the fourth section – the program successfully discriminates between convex and concave surfaces. Recent evidence\textsuperscript{11} suggests that the human visual system may be capable of convex/concave discrimination by means of a similar test. In the fifth section it is shown that extended viewer

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image and vision computing
movements (where the specular point is tracked through many images) constrain the reflecting surface to coincide with one member of a one-parameter family of curves. If the curve passes through a known point on the surface, then the ambiguity is removed and the curve uniquely determined.

**MOTION OF SPECULAR POINTS**

Koenderink and van Doorn⁹ give a qualitative description of the movement of specularities as the vantage point changes by considering the Gauss map of the surface. Using this analysis it is clear that the velocity of the specularity is less if the curvature is high (it depends on the Weingarten map of the surface – the differential of the Gauss map) so that 'specularities' tend to cling to the strongly curved parts; and also that specularities are created or annihilated in pairs at parabolic lines on the surface and move transversely to the lines at their moment of creation. This approach is valid provided the distance of the viewer from the surface is much greater than the largest radius of curvature.

Local metric information (constraints on the surface curvature) can be obtained from two views⁹ provided the position of the light source and a surface feature (close to the reflecting point) are known. The two views might be a stereo pair or from a moving monocular observer. In either case the baseline is assumed known and surface features can be matched using the epipolar constraint. The specularity will move relative to these surface features. The constraints on surface curvature are contained in the specular motion equation⁹. This is a linearized relation between the change \( \mathbf{r} = (r_1, r_2, r_3) \) in the position of the specularity in the tangent plane of the surface and the (small) viewer movement \( \mathbf{d} = (d_1, d_2, d_3) \). The coordinate frame is chosen to be the local normal frame where the origin lies on the surface (at the reflecting point) and the z-axis is along the local surface normal. It is arranged so incident and reflected rays are in the \( xy \) plane and the movement of the specularity is in the \( xy \) plane – the local tangent plane. The vector \( \mathbf{x} = (x, y) \) is the restriction of \( \mathbf{r} = (x, y, z) \) to the tangent plane. The linear system can be expressed as:

\[
2V(M - k_{vl}I)x = w,
\]

where

\[
\mathbf{w} = (-d_1 + d_3 \tan \sigma, -d_3, -d_2)^T,
\]

\[
M = \begin{pmatrix}
  \sec \sigma & 0 \\
  0 & \cos \sigma
\end{pmatrix},
\]

and

\[
k_{vl} = \frac{1}{2} \left( \frac{1}{V} + \frac{1}{L} \right)
\]

The angle of reflection is \( \sigma \). Vectors \( \mathbf{V} \) and \( \mathbf{L} \) are vectors from the origin to the viewer and light source (\( V = ||\mathbf{V}|| \) and \( L = ||\mathbf{L}|| \)). The Hessian matrix \( \mathbf{H} \) is the matrix of second partial derivatives of the surface \( z(x, y) \).

In the normal coordinate frame the eigen-values of \( \mathbf{H} \) are the principal curvatures of the surface.

This linear approximation is valid if the baseline is relatively short, that is, when

\[
||\mathbf{d}|| \ll ||\mathbf{V}|| \cos \sigma,
\]

and provided the surface does not focus incoming rays to a point or line close to the centre of projection.

**LOCAL VIEWER MOVEMENT**

A simple test is described, making minimal assumptions, for distinguishing between convex and concave surfaces. Loosely, it is shown that on a convex surface the specularity moves with the viewer (sympathetic motion) whereas on a concave surface the movement is (in general) against the viewer motion (contrary motion). One can convince oneself that this is true by looking at specular reflections in the front and back surface of a spoon. The terms 'moves with' and 'moves against' the viewer motion are made precise in the following theorem (which is proved in the appendix).

**Theorem 1**

If \( \mathbf{H} \) is negative definite (surface locally convex elliptic) then \( \mathbf{d}_\perp \cdot \mathbf{r}_\perp \geq 0 \). If \( \mathbf{H} \) is positive definite (surface locally concave elliptic) and the smallest principal curvature \( k \) satisfies \( k > \sec \sigma \gamma \) then \( \mathbf{d}_\perp \cdot \mathbf{r}_\perp \leq 0 \).

Here, \( \mathbf{d}_\perp \) and \( \mathbf{r}_\perp \) are the projections of the vectors \( \mathbf{d} \) and \( \mathbf{r} \) onto the plane perpendicular to \( \mathbf{V} \). Their calculation is described below. The corollaries provide useful tests because other surface shapes are possible (for example hyperbolic) where the scalar product could have either sign.

If \( \mathbf{d}_\perp \cdot \mathbf{r}_\perp < 0 \) (contrary motion) then the surface is not convex.

If \( \mathbf{d}_\perp \cdot \mathbf{r}_\perp > 0 \) (sympathetic motion) then the surface is not concave unless one of the principal curvatures is less than \( \sec \sigma \gamma \).

The first test shows it is always possible to determine if a surface is not convex.

**Calculation of \( \mathbf{d}_\perp \cdot \mathbf{r}_\perp \)**

The geometry is shown in Figure 1. The simple vector cycle is used:

\[
\mathbf{V} + \mathbf{d} - \mathbf{W} - \mathbf{r} = 0
\]

The projected vectors are

\[
\mathbf{d}_\perp = \mathbf{d} - (\mathbf{d} \cdot \mathbf{V})\mathbf{V}
\]

\[
\mathbf{r}_\perp = \mathbf{d}_\perp - \mathbf{W}_\perp
\]

\[
= \mathbf{d}_\perp - \mathbf{WU}
\]

where \( \mathbf{U} = \mathbf{W} - (\mathbf{W} \cdot \mathbf{V})\mathbf{V} \), and \( \mathbf{V} \) indicates a unit vector. Hence,

\[
\mathbf{d}_\perp \cdot \mathbf{r}_\perp = ||\mathbf{d}_\perp||^2 - W(\mathbf{d}_\perp \cdot \mathbf{U})
\]

(3)
Figure 1. Specularity geometry. The viewer moves between points C and D. The specularity moves (in the tangent plane of the surface) between A and B. The vectors \( \mathbf{V} \) and \( \mathbf{W} \) are then reflected rays at A and B.

Figure 2. Projected vectors in the plane perpendicular to \( \mathbf{V} \). It is clear that if the magnitude of \( \mathbf{W} \) (and hence \( \mathbf{W}_f \)) varies, the scalar product \( \mathbf{d}_v \cdot r_1 \) can have either sign.

To calculate the scalar product involves only knowing the viewer motion \( \mathbf{d}_v \), the directions of the reflected ray in each view \( \mathbf{V} \) and \( \mathbf{W} \) and an estimate of the surface distance \( W \). The important point is that the test does not require any knowledge of the light source position or the surface slant. The estimate of the viewer-surface distance can be obtained, for example, from a nearby surface feature (whose position can be measured using binocular stereo). It is worth noting that in the coordinate frame defined by the triad \( \{ \mathbf{d}_v, \mathbf{V}, \mathbf{W} \} \) the length \( \mathbf{d}_v \cdot r_1 \) is the epipolar (horizontal) disparity.

**ERRORS IN CONVEX/CONCAVE TEST**

The test only involves the sign of the scalar product. However, the magnitude can be used to gauge the immunity to errors.

The important question is how sensitive is the sign of the scalar product to the estimate of \( W \)? It is clear from equation (2) and Figure 2 that the scalar product will *always* change sign eventually as \( W \) varies. From (3) the sign change occurs at \( W_0 \), where

\[
W_0 = \frac{||\mathbf{d}_0||^2}{\mathbf{d}_v \cdot \mathbf{U}}
\]

or

\[
W_0 = \frac{||\mathbf{d}_0||^2}{\mathbf{W} \cdot \mathbf{d}_v}
\]

The error in this estimate is given by

\[
\delta W = \frac{\partial W_0}{\partial \mathbf{V}} \cdot \delta \mathbf{V} + \frac{\partial W_0}{\partial \mathbf{W}} \cdot \delta \mathbf{W}
\]

where \( \delta \mathbf{V} = \) (error in \( \mathbf{V} \) perpendicular to \( \hat{\mathbf{V}} \))||\( \mathbf{V} \)|| and it is assumed that the error in \( \mathbf{d}_v \) is negligible. Provided the estimate of \( W \) is outside the range \( (W_0 \pm \delta W) \) one can be confident in the sign of the scalar product.

**Results of convex/concave test**

The results of applying this test to the image pairs shown in Figures 3-5 are tabulated in Table 1.

For the computation, the specularities are detected using Brelstaff's specularity detector and matcher\(^4\); the estimate of \( W \) is obtained from the distance to a surface feature close to the specularity. This distance is determined by the PMF binocular stereo algorithm\(^5\). The error estimates for \( \mathbf{V} \) and \( \mathbf{W} \) are based on an error of \( \pm \frac{1}{2} \) pixel in localizing the brightest point of the specularity in the image. In all cases (see Table 2) the estimate of \( W \) falls outside the region \( (W_0 \pm \delta W) \) so one can be confident that the sign of the scalar product is correct.
Table 1. Scalar product and interpretations

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>$d_x, r_x$</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>convex ellipsoid</td>
<td>0.024991</td>
<td>not concave</td>
</tr>
<tr>
<td>4</td>
<td>concave ellipsoid</td>
<td>0.021369</td>
<td>not convex</td>
</tr>
<tr>
<td>5</td>
<td>beach ball</td>
<td>0.001497</td>
<td>not concave</td>
</tr>
</tbody>
</table>

Table 2. Viewer-surface distance estimate and safety margins

<table>
<thead>
<tr>
<th>Figure</th>
<th>$W$</th>
<th>$W_0$</th>
<th>$\delta W_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>10.574994</td>
<td>10.799026</td>
<td>0.037323</td>
</tr>
<tr>
<td>4</td>
<td>10.632490</td>
<td>10.447312</td>
<td>0.028527</td>
</tr>
<tr>
<td>5</td>
<td>0.900474</td>
<td>0.964750</td>
<td>0.000453</td>
</tr>
</tbody>
</table>

The first two pairs of stereo images (Figures 3 and 4) are computer generated using a narrow field of view to exaggerate disparities. This makes it easy to see relative displacement of specularities. Figure 3 shows a convex surface with a stereoscopically visible specularly and near surface markings. Note that the displacement of the specularly (relative to surface markings) is in the same direction as the relative displacement of the optical centres of left and right views, i.e. the motion of the specularly is sympathetic ($d_x, r_x > 0$). Figure 4 shows a concave surface and, as expected, the relative displacement of the specularly is reversed, opposing the relative displacement of the motion of the viewer ($d_x, r_x < 0$). Figure 5 is a real image of a beach ball (convex) and again the motion of the specularly is sympathetic ($d_x, r_x > 0$).

GLOBAL VIEWER MOVEMENT

This section describes what information is available from extended or continuous viewer movements where the information from many views is combined.

The following theorem is proved:

**Theorem 2**

Given a source of light $S$ (or a direction of light from infinity), and for each point $r$ on a curve in $R^3$, given the direction of a reflected ray through $r$ (the reflection being from an unknown reflecting surface $M$) then this determines a unique curve $m$ (the locus of the reflecting points) on $M$ provided one point on $m$ is known.

The proof is given in the appendix.

Thus, given a fixed light source and surface, and observing the direction of the reflected rays as the viewer moves, uniquely determines a curve on the reflecting surface. Without a known point $r$ there is ambiguity as the curve is only determined by the directions of the reflected rays to lie in a one parameter family. If a point is known on the curve, the ambiguity is removed and the curve uniquely determined. The required point $r$ could be found where the curve $m$ crosses an edge whose position is known from binocular stereo.

The surface normal is also known along $m$. This places strong constraints on the local surface curvature. Furthermore, by making a second movement that crosses the original path (so that the paths are transverse and both transverse to the reflected rays) transverse curves are obtained on the reflecting surface. At the point where these curves cross the surface curvature (principal curvatures and direction of principal axis) is completely determined. The type and extent of the constraints placed on the surface by curves of this type are currently being explored.

REFERENCES

8. Buchanan, C S ‘Determining surface orientation from specular highlights’ *MSc Thesis* Department of Computer Science, University of Toronto, Canada (1987)
ACKNOWLEDGEMENTS

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APPENDIX

PROOF OF THEOREM 1

The vectors $r_1$ and $d_1$ lie in the plane perpendicular to $V$. The scalar product $d_1, r_1$ is calculated in the coordinate frame with $z$-axis along $V$ and $y$-axis parallel to the $y$-axis of the normal frame. In this frame the vectors $r_1$ and $d_1$ have zero $x$ components. Their $x$ and $y$ components are obtained from $x$ and $d$ using the projection matrix $P^{-1}$, where

$$
P = \begin{pmatrix}
\sec \sigma & 0 \\
0 & 1
\end{pmatrix}
\quad \text{and} \quad
P^{-1} = \begin{pmatrix}
\cos \sigma & 0 \\
0 & 1
\end{pmatrix}
$$

Then

$$
d_\perp = \begin{pmatrix}
P^{-1}x \\
0
\end{pmatrix}
\quad \text{and} \quad
P^{-1}d_\perp = \begin{pmatrix}
-P^{-1}w \\
0
\end{pmatrix}
$$

Using the specular motion equation (1)

$$
P^{-1}V(MH) - k_{vl}I)P(P^{-1}x) = P^{-1}w
$$

and hence

$$
d_\perp, r_\perp = -(P^{-1}x)^T P^{-1}w
$$

$$
= -2V(P^{-1}x)^T [P^{-1}(MH - k_{vl}I)P](P^{-1}x)
$$

This is a quadratic form

$$
d_\perp, r_\perp = -2V(P^{-1}x)^T Q(P^{-1}x)
$$

The sign depends on the eigen-values of $Q$. Noting that $M = \cos \sigma P^2$

$$
Q = \cos \sigma P HP - k_{vl}I
$$

Now, det $PHP = \sec^2 \sigma \det (H)$, and considering the trace of $PHP$ or noting that $(0,1)$ PHP $(0,1)^T = (0,1)$ H $(0,1)$, proves PHP is positive (negative) definite as H is positive (negative) definite.

Two cases are considered:

1. $H$ negative definite
   $Q$ is negative definite, hence $d_\perp, r_\perp \geq 0$.

2. $H$ positive definite
   A lower bound on the smallest eigen-value of PHP is given by $k_1$, where $k_1$ is the smaller eigen-value of $H$. $Q$ will have a negative eigen-value if an eigen-value of PHP is less than $\sec \sigma k_{vl}$. However, provided $k_1 > \sec \sigma k_{vl}$, $Q$ is positive definite and $d_\perp, r_\perp \leq 0$.

PROOF OF THEOREM 2

The reflected rays are all normal to the orthotomic $W$ of $M$ relative to $S$ (the orthotomic trial is the locus of reflections of $S$ in tangent planes to $M$).

Choose any point $q$ on the reflected ray (see Figure 6). There is a unique piece of curve $w_1$ through $q$ perpendicular to all the reflected rays through points $p$. This curve $w_1$ is on a possible orthotomic surface $W_1$ through $q$.

Taking perpendicular bisector planes of segments joining $S$ to points $t$ of $w_1$ gives a one-parameter family of planes which are tangent to a possible mirror $M_1$. For each $t$ the intersection of the perpendicular bisector plane with the reflected ray through $t$ determines a point on $M_1$.

Hence, the choice of $q$ determines a curve $m_1$ on a possible mirror $M_1$. Changing $q$ will change $w_1$ and hence $m_1$, so $q$ can be adjusted until $m_1$ passes through a known point on $M$. Thus, provided such a known point exists the curve and the surface normals along the curve are determined.

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1The proof is slightly modified if the light source is at infinity