Beyond Connecting the Dots: Polynomial-time algorithm for Segmentation and Boundary Estimation with Imprecise User Input

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The Problem

**Given:** An image and a set of points describing the object boundary

Image $\mathcal{I} : \Omega \rightarrow \mathbb{R}$

Input points $X_0, \ldots, X_N \in \mathbb{R}^2$
The problem

Find: An edge based segmentation through the vicinity of the input points

Curve $C : [0, \mathcal{L}(C)] \rightarrow \mathbb{R}^2$
(parameterized by arc length)
Notation

- \( I : \Omega \rightarrow \mathbb{R} \) denote the image (gray scale).
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- $X_1, \ldots, X_K \in \Omega$ be the user input points on the boundary.
- $I : \Omega \rightarrow \mathbb{R}$ denote the image (gray scale).
- $X_1, \ldots, X_K \in \Omega$ be the user input points on the boundary.
- $C : [0, \mathcal{L}(C)] \rightarrow \Omega$ be a parameterised curve. So $C(0)$ is the beginning of the curve, $C(\mathcal{L}(C))$ is the end point of the curve.
Define the distance of point $X_i$ from the curve $C$ as:

$$d(C,X_i) = \min_{s \in [0, L(C)]} \| X_i - C(s) \|$$
Cost Function

\[ E(C) = \alpha \sum_{i=1}^{K} d(C, X_i) + \int_{0}^{L(C)} \left[ g(C(s)) + \lambda |\kappa_C(s)|^2 \right] ds. \]

is NP-hard to optimize.
Rewriting Cost Function

\[
E(C) = \min_{C \in \mathcal{C}, s_0, \ldots, s_N \in [0, \mathcal{L}(C)]} \alpha \cdot \sum_{i=0}^{N} d(C(s_i), X_i) + \int_{0}^{\mathcal{L}(C)} \left[ g(C(s)) + \lambda \cdot \kappa C(s)^2 \right] ds
\]

distance term

same energy function, written differently.
Ordered points

$$\min_{C \in \mathcal{C}, s_0, \ldots, s_N \in [0, \mathcal{L}(C)]} E(C) = \alpha \cdot \sum_{i=0}^{N} d(C(s_i), X_i) + \int_{0}^{\mathcal{L}(C)} \left[ g(C(s)) + \lambda \cdot \kappa C(s)^2 \right] ds$$

s.t. \quad s_0 \leq s_1 \leq \ldots \leq s_N.$$

is tractable!
Minimizing this Energy

\[ E(C, s_1, \ldots, s_K) = \alpha \sum_{i=1}^{K} \| C(s_i) - X_i \| + \int_{0}^{\mathcal{L}(C)} g(C(s)) \, ds \]
Graph Structure

\[ E(C, s_1, \ldots, s_K) = \alpha \sum_{i=1}^{K} \| C(s_i) - X_i \| + \int_{0}^{\mathcal{L}(C)} g(C(s)) \, ds \]
Results

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[Images of tiger and human shapes with outlined contours and markers]