1. Lecture 1: The camera as a geometric device

(a) First find the rotation from World to aligned frame A (either in steps — or in one go here as it is obvious). Second handle the translation.

\[
    X_c = \begin{bmatrix}
    0 & 1 & 0 & 0 \\
    0 & 0 & 1 & -h \\
    1 & 0 & 0 & 4h \\
    0 & 0 & 0 & 1
\end{bmatrix} X_w
\]

(b) Either project then let \( t \to \infty \) or vice versa. You should find

\[
    x = \begin{bmatrix}
    1/2 \\
    3/4 \\
    1
\end{bmatrix}
\]

(c) All in the lecture notes.

(d) \([x, y] = [750, 790]\)

2. Lecture 2: Camera calibration

(a) You need to be able to reproduce each step in the method from the lecture notes.

(b) The code in calibration.m also fixes some sign ambiguities which were mentioned but not explained in the lecture (they would not be asked for in June). They arise because you can change the sign of a row or column of a rotation matrix and it remains a rotation matrix. To understand them, write the product KR in terms of symbols

\[
    KR = \begin{bmatrix}
    (f R_{11} + s R_{21} + u_0 R_{31}) & (f R_{12} + s R_{22} + u_0 R_{32}) & (f R_{13} + s R_{23} + u_0 R_{33}) \\
    (\gamma f R_{21} + v_0 R_{31}) & (\gamma f R_{22} + v_0 R_{32}) & (\gamma f R_{23} + v_0 R_{33}) \\
    K_{33} R_{31} & K_{33} R_{32} & K_{33} R_{33}
    \end{bmatrix}
\]

and argue that \( f, \gamma f \) and \( K_{33} \) must be positive. So

- if \((f < 0)\), change the signs of \( f, R_{11}, R_{12}, R_{13} \).
- if \((\gamma f < 0)\), change the signs of \( \gamma f, s, R_{21}, R_{22}, R_{23} \).
- if \((K_{33} < 0)\), change the signs of \( K_{33}, u_0, v_0, R_{31}, R_{32}, R_{33} \).
(c) Projection matrix is (up to scale)

\[
\begin{bmatrix}
2 & 1 & 0 & 8 \\
1 & 0 & 2 & 2 \\
1 & 0 & 0 & 4
\end{bmatrix}
\]

and non-homogenous points are

\[
\begin{bmatrix}
x \\
y
\end{bmatrix} = \begin{bmatrix} 2 \\ 1 / 2 \end{bmatrix}, \begin{bmatrix} 9 / 4 \\ 1 / 2 \end{bmatrix}, \begin{bmatrix} 9 / 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 11 / 5 \\ 1 \end{bmatrix}
\]

(d) Results from Matlab

\[
\text{intrinsic} = \\
\begin{bmatrix}
1.0000 & 0.0000 & 2.0000 \\
0 & 2.0000 & 1.0000 \\
0 & 0 & 1.0000
\end{bmatrix}
\]

\[
\text{Rot} = \\
\begin{bmatrix}
0.0000 & 1.0000 & 0.0000 \\
-0.0000 & -0.0000 & 1.0000 \\
1.0000 & -0.0000 & 0.0000
\end{bmatrix}
\]

\[
\text{translation} = \\
\begin{bmatrix}
0.0000 \\
-1.0000 \\
4.0000
\end{bmatrix}
\]

3. **Lecture 2: Edges, Corners and SIFTs**

(a) i) Try distorting a vertical line \(x=D\), say, and sketching the result.
ii) This is straightforward and involves neither small quantities, expansions, or approximation.
iii) Think of man-made environments and parts of Lecture 2.

(b) Straightforward

(c) Practice explaining the method and mathematics.

(d) Ditto

(e) Ditto

(f) Ditto

4. **Lecture 3: Epipolar Geometry**

(a) Although the notes are short they should be full of information.

(b) Practice the derivation of \(F\)

(c) Ditto

(d) See course 4.
5. Lecture 3: Epipolar Geometry

(a) i) Should find
\[
R = \begin{bmatrix}
\frac{1}{2} & 0 & -\frac{\sqrt{3}}{2} \\
0 & 1 & 0 \\
\frac{\sqrt{3}}{2} & 0 & \frac{1}{2}
\end{bmatrix} \\
t = d[\sqrt{3}/2, 0, 1/2]^T \\
K = K' = I
\]

\[
F = \begin{bmatrix}
0 & (-1/2)d & 0 \\
(-1/2)d & 0 & (-\sqrt{3}/2)d \\
0 & (\sqrt{3}/2)d & 0
\end{bmatrix}
\]

ii)
\[
I' = \begin{bmatrix}
1 \\
1 + \frac{1}{\sqrt{3}} \\
-\frac{1}{\sqrt{3}}
\end{bmatrix}
\]

(b) i) Optical centres of C and C' are
\[
P = \begin{bmatrix}
0 \\
1
\end{bmatrix} \\
-\begin{bmatrix}
R^T \\
t
\end{bmatrix}
\]

ii) Epipoles are
\[
e = KR^Tt \\
e' = K't
\]

iii) The right and left null spaces test for \(Fe = 0\) and \(F^Te' = 0\), respectively, and remember that \([t]_x\) is skew-symmetric.

What is the relevance of the result? It is that a feature at the epipole in one image MUST be imaged and the epipole in the other image. It is the point for which the epipolar line collapses to a point. One way of seeing this is that the epipolar line \(l' = Fe = 0\). Now any epipolar line \(l'\) can be described by two points \(l' = e' \times a\). To make this zero, \(a = e'\).

iv)

(c) i) Straightforward!
\[
x' = \begin{bmatrix}
0 \\
1
\end{bmatrix} \\
-\begin{bmatrix}
R^T \\
t
\end{bmatrix}
\]

\[x' = K'RK_x = Hx\]

The matrix \(H\) is a 3 \times 3 matrix with 8 d.o.f.

ii) Draw a diagram! Think about a square pixel in the transformed image and how its corners would appear in the original.

6. Lecture 4: Dense stereo correspondence Straightforward

7. Lecture 4: RANSAC Remember your probability tutorials.

8. Lecture 4: Triangulation

(a) Straightforward

(b) By construction it is easy to see that
\[
X_i = [-2, 0, 2f, 1]^T \\
X_{ii} = [0, 0, 1, 0]^T
\]

(c) Straightforward